

ENGINEERINGTHERMODYNAMICS
(ME3391)

UNIT I BASICS, ZERO TH AND FIRST LAW

Review of Basics – Thermodynamics systems, Properties and processes

Thermodynamic Equilibrium - Displacement work - P-V diagram.

Thermal equilibrium - Zeroth law – Concept of temperature and Temperature Scales. First law – application to closed and open systems – steady and unsteady flow processes.

UNIT II SECOND LAW AND ENTROPY

Heat Engine – Refrigerator - Heat pump. Statements of second law and their equivalence & corollaries. Carnot cycle - Reversed Carnot cycle - Performance - Clausius inequality. Concept of entropy - T-s diagram - Tds Equations - Entropy change for a pure substance.

UNIT III AVAILABILITY AND APPLICATIONS OF II LAW

Ideal gases undergoing different processes - principle of increase in entropy. Applications of II Law. High and low grade energy. Availability and Irreversibility for open and closed system processes - I and II law Efficiency

UNIT IV PROPERTIES OF PURE SUBSTANCES

Steam - formation and its thermodynamic properties - p-v, p-T, T-v, T-s, h-s diagrams. P-v-T surface. Determination of dryness fraction. Calculation of work done and heat transfer in non-flow and flow processes using Steam Table and Mollier Chart.

UNIT V GAS MIXTURES AND THERMODYNAMIC RELATIONS

Properties of Ideal gas, real gas - comparison. Equation of state for ideal and real gases. Vander Waal's relation - Reduced properties - Compressibility factor - Principle of Corresponding states - Generalized

Compressibility Chart. Maxwell relations - TdS Equations - heat capacities relations - Energy equation, Joule-Thomson experiment - Clausius-Clapeyron equation.

UNIT I

BASICS, ZERO TH AND FIRST LAW

Review of Basics – Thermodynamic systems, Properties and processes
Thermodynamic Equilibrium - Displacement work - P-V diagram. Thermal equilibrium - Zeroth law – Concept of temperature and Temperature Scales. First law – application to closed and open systems – steady and unsteady flow processes.

Nomenclature

A	area
C	velocity
$^{\circ}C$	temperature on the Celsius (or Centigrade) scale
c	specific heat
c_p	specific heat at constant pressure
C_v	specific heat at constant volume
C_p	molar heat at constant pressure
C_v	molar heat at constant volume
D, d	bore; diameter
H	enthalpy
h	specific enthalpy; heat transfer coefficient
P	power
N	rotational speed
K	temperature on Kelvin scale
L	stroke
M	molecular weight
m	mass
Q	heat, rate of heat transfer
q	rate of heat transfer per unit area
R	gas constant
R_0	universal gas constant
R	radius, expansion ratio, compression ratio

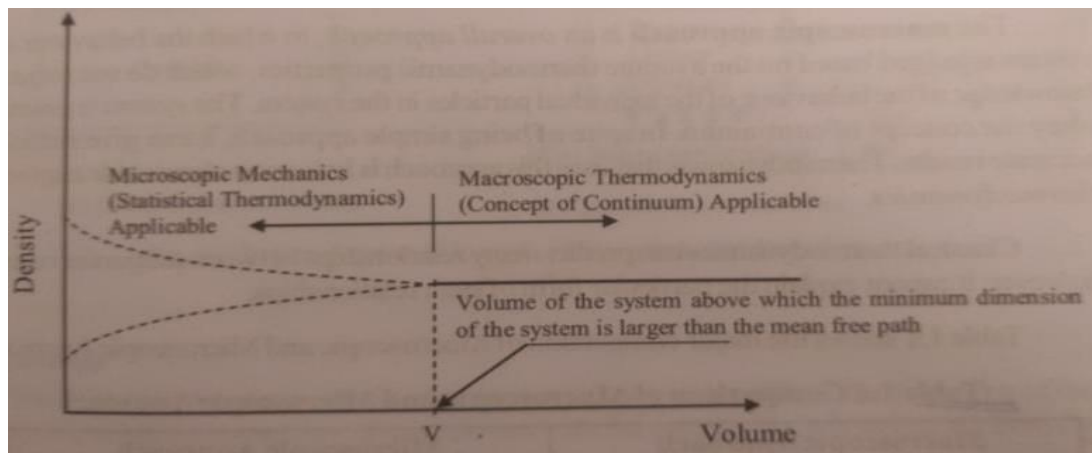
S	entropy
T	absolutetemperature;torque
U	internalenergy;overallheattransferco-efficient
u	specificinternalenergy
V	volume
v	specificvolume
W	work;rateofworktransfer;brakeload; weight
w	specificweight;velocityofwhirl
x	drynessfraction;lengthtivepressure,backpressure

Symbols α absorptivity γ ratioofspecificeats, $\frac{c_p}{c_v}$ ϵ emissivity;effectiveness η efficiency θ temperaturredifference,angle ρ density σ Stefan-Boltzmannconstant ϕ relative humidity, angle.	Universalgasconstant, $R_u=8.314$ kJ/kmole – K Characteristicgasconstant, c $R_c = \frac{RU}{M}$ ForAir $R=8.314 \times \frac{1}{29} = 0.287$ kJ/kg-K Forwater $R=8.314 \times \frac{1}{18} = 0.461$ kJ/kg-K UnitsofheatandworkiskJ UnitsofpressureisN/m ² 1 kPa= 10 ³ Pa 1bar=1×10 ⁵ N/m ² 1bar=1×10 ³ kN/m ²
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Thermodynamics is science which deals with the relations among heat, work and properties of system which are in equilibrium.

Thermodynamics=Thermo +dynamics
(Heat) (Motion)

concept of continuum is an a area that can keep being divided and divided infinitely ; no individual particles. It is a simplification that makes it possible to investigate the movement of matter on scales larger than the distance between particles.



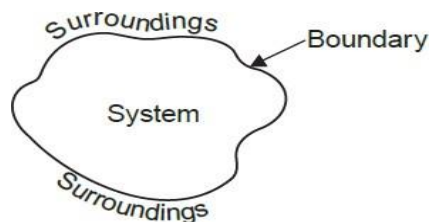
Concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables.

THERMODYNAMICS SYSTEMS

System, Boundary and Surroundings

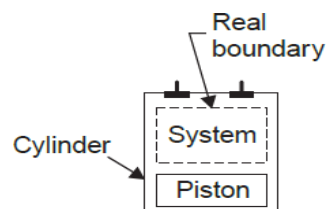
System:

A system is a finite quantity of matter



Boundary.

The *actual envelope enclosing the system* is the boundary of the system. The boundary may be fixed or it may move, as and when a system containing a gas is compressed or expanded.



Surroundings contain everything other than the system.

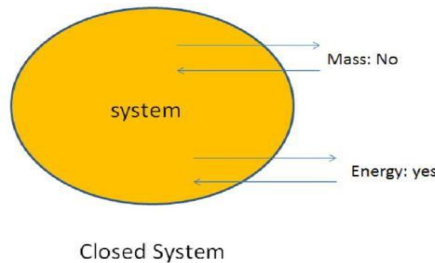
Universe: A system and surrounding together comprise a universe

Type of thermodynamic systems

1. Closed System:
2. Open System
3. Isolated System

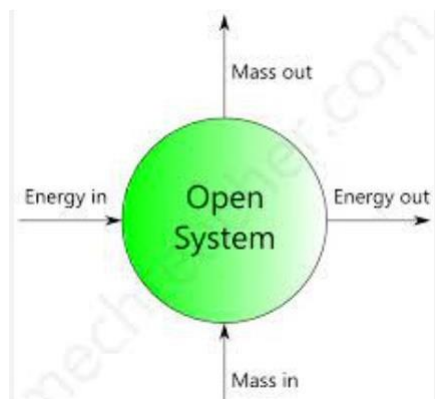
Closed System:

System does not permit any mass transfer. But only the energy transfer takes place.



Open System

An **open system** both mass and energy transfer take place.



Isolated System: An isolated system, which is not affected by surrounding is called isolated system.



Adiabatic System

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Homogeneous System

A system which consists of a single phase is termed as homogeneous system. Examples: Mixture of air and water vapour, water plus nitric acid and octane plus heptane.

Heterogeneous System

A system which consists of two or more phases is called a heterogeneous system. Examples: Water plus steam, ice plus water and water plus oil.

Thermodynamic property and State (2 marks)

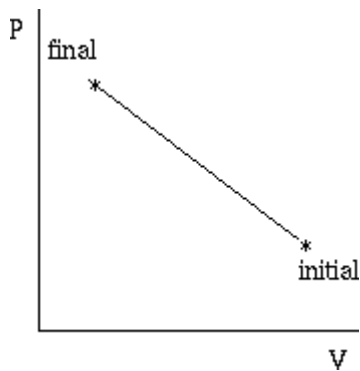
Thermodynamic property Any observable characteristics required to describe the conditions or state of a system is known as Thermodynamic property of a system.

Differentiate Intensive and Extensive Property

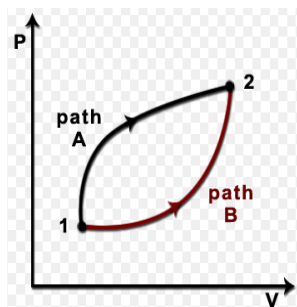
	Extensive properties	Intensive properties
mass of a system	dependent on	independent of
additive.	yes, additive.	Not additive.
Example:	mass (m), volume (V), Energy (E), Enthalpy (H)	Example: Pressure (P), Temperature (T), Density

.State: Is the condition of the system at any particular moment.

- It is the condition of a system as defined by the values of all its properties.
- It gives a complete description of the system.
- Any operation in which one or more properties of a system change is called change of state.



Path: A thermodynamic property that depends on the path between the initial and final state is known as the path function. The path functions depend on the path taken or covered between two (initial and final) states.



Phase. A phase is a quantity of matter which is homogeneous throughout in chemical composition and physical structure

MACROSCOPIC AND MICROSCOPIC POINTS OF VIEW

1. Macroscopic approach—(Macro means big or total)
2. Microscopic approach—(Micro means small)

<i>Detail</i>	<i>Macroscopic approach</i>	<i>Microscopic approach</i>
approach	concerned with overall behaviour.	structure of the matter
analysis	mathematical formulae.	statistical methods
values of the properties	average values.	changes in properties
describe a system	few properties are needed.	Large number of variables

THERMODYNAMIC EQUILIBRIUM

A system is in **thermodynamic equilibrium** if the temperature and pressure at all points are the same; there should be no velocity gradient;

1. **Thermal equilibrium.** The temperature of the system does not change with time and has the same value at all points of the system.

2. **Mechanical equilibrium.** There are no unbalanced forces within the system or between the surroundings. The pressure in the system is the same at all points and does not change with respect to time.

3. **Chemical equilibrium.** No chemical reaction takes place in the system and the chemical composition which is the same throughout the system does not vary with time.

THERMODYNAMIC PROCESS

Path and Process:

The series of states through which a system passes during a change of state is the path of the system.

PROCESS: change of state undergone by a system from one equilibrium state to another state.

We can allow one of the properties to remain constant during a process.

Isothermal Constant Temperature (T)

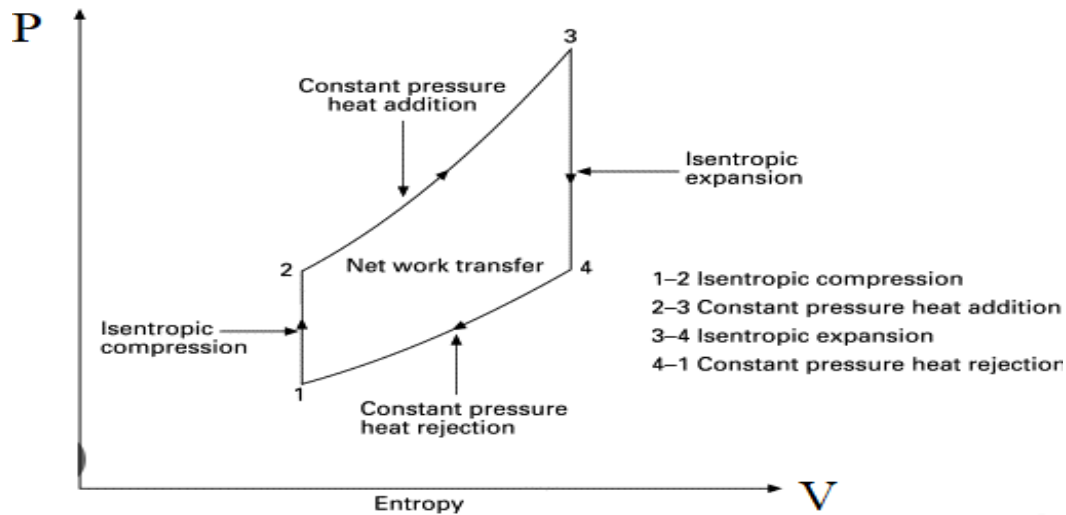
Isobaric Constant Pressure (P)

Isochoric Constant Volume (V)

Isentropic Constant Entropy (s)

Isenthalpic Constant Enthalpy (h)

Process occurs: an energy transfer at a steady state.



A process may be

i) **non-flow process**: non-flow in which a fixed mass within the defined boundary is undergoing a change of state.

Example: A substance which is being heated in a closed cylinder undergoes

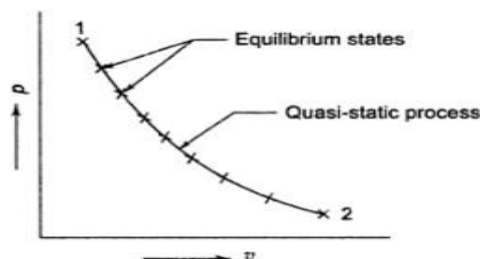
ii) **flow process** mass is entering and leaving through the boundary of an open system.

quasi-static process

A **quasi-static process** is one in which the deviation from thermodynamic equilibrium is infinitesimal.

Characteristics:

- referred to as a slow process.
- happens at an infinitesimally slow rate,
- has a lot of static equilibrium.
- thermodynamic equilibrium with its surroundings at all times



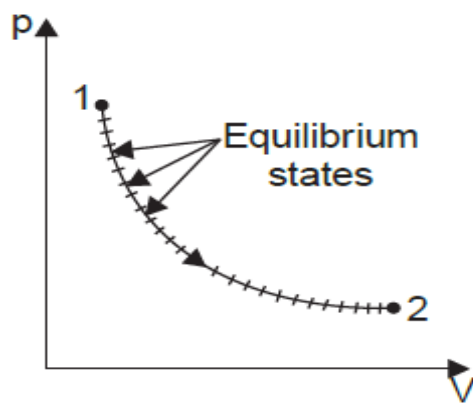
Non-Quasi-static process.

If a process is carried out in such a way that at every instant the system departs finitely from thermodynamic equilibrium

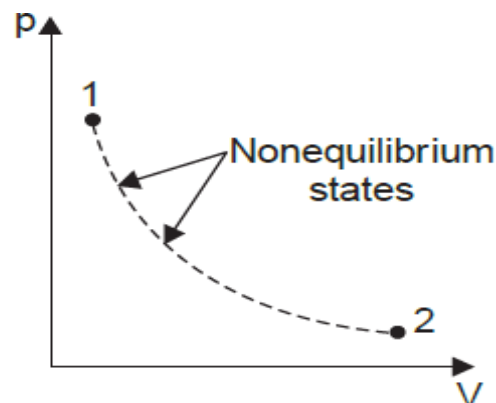
REVERSIBLE and IRREVERSIBLE PROCESSES:

Reversible process. A reversible process (also some times known as quasi-static process) is one which can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states. This process has the following characteristics:

1. It must pass through the same states on the reversed path as were initially visited on the forward path.
2. This process when undone will leave no history of events in the surroundings.
3. It must pass through a continuous series of equilibrium states.



Reversible process.



Irreversible process.

Examples.

- (i) Frictionless relative motion.
- (ii) Expansion and compression of a spring.

Irreversible process.

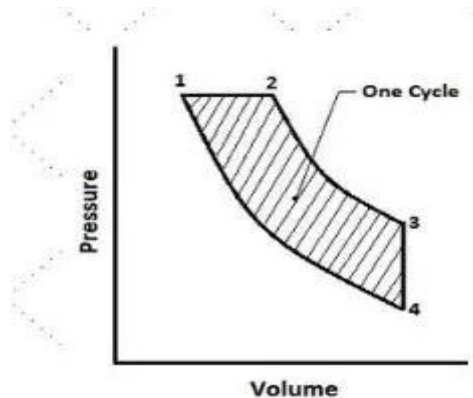
An irreversible process is one in which heat is transferred through a finite temperature.

Examples.

- (i) Relative motion with friction
- (ii) Combustion
- (iii) Diffusion
- (iv) Free expansion
- (v) Throttling
- (vi) Electricity flow

CYCLE

Thermodynamic cycle refers to number of processes in sequence bring back the system to its original volume, then the system is said to be a **cycle**.



In fig a system starts from state 1 and then undergoes (1-2), (2-3), (3-4), (4-1)

And it returns back to its original position.

	Point Function	Path Function
change of the path	is independent	depends on the path
magnitude of such quantity	depends on the state.	area under the curve
	These are exact differential.	inexact differential.
Ex:	like pressure (P), volume (V), Temp. (T),	Heat and work

ENERGY, WORK AND HEAT

Energy: Energy is a general term embracing energy in transition and stored energy.

Work

Work is said to be done when a force moves through a distance.

$$W = F \times x \quad \text{N-m}$$

Sign convention :

_ If the work is done by the system on the surroundings, e.g., when a fluid expands pushing a piston outwards, the work is said to be *positive*.

$$\text{Work output of the system} = +W$$

_ If the work is done on the system by the surroundings, e.g., when a force is applied to a rotating handle, or to a piston to compress a fluid, the work is said to be *negative*.

$$\text{Work input to system} = -W$$

Heat is the transfer of thermal energy between systems, while work is the transfer of mechanical energy between two systems

Heat is defined as a form of energy that can be transferred from one system to another but there must be a temperature difference between these two systems.

Heat is usually represented by 'Q' in (kJ)

$$\int_1^2 \delta Q = Q_{1-2}$$

Sign convention :

Heat received by the system = +Q

Heat rejected or given up by the system = -Q.

	Heat	Work
Grade of energy	poor	high
source	Produced anywhere	requires
Converted into works	friction	fully
Motion of molecules	Random	ordered
States	Form of energy	Transferring energy.

Zeroth law of thermodynamics' states that if two systems are each equal in temperature to a third, they are equal in temperature to each other.

$$a = b \text{ and } b = c, \text{ then } a = c.$$

Application of First Law to Non-flow/Closed system

1. Constant volume process

2. Constant pressure process
3. Constant temperature process
4. Reversible adiabatic process
5. Polytropic process

1. Constant volume process

If heat is supplied at constant volume then the pressure and temperature of system will increase but there is no any expansion or compression of gas

Work done $dW = 0$

General gas equation $\frac{pV}{T} = C$

p, V and T relations $\frac{p_1}{T_1} = \frac{p_2}{T_2}$

Heat supplied to the system $= mc_v(T_2 - T_1)$

Change in internal energy (Change in enthalpy) $du = dQ = m c_v(T_2 - T_1)$

Change in entropy

$\Delta S = mc_v \ln \left(\frac{T_2}{T_1} \right)$

Constant pressure or isobaric process

General gas equation $\frac{pV}{T} = C$

p, V and T relations $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Heat supplied to the system $= mc_p(T_2 - T_1)$

Work done $dW = p (V_2 - V_1)$

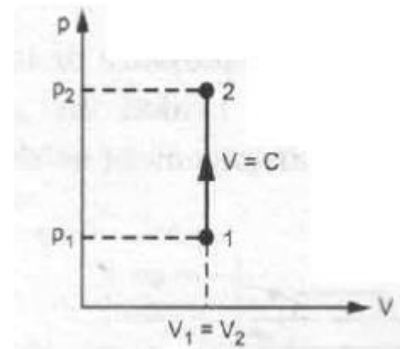
Change in internal energy

(Change in enthalpy) $dh = dQ = m c_p(T_2 - T_1)$

Change in entropy

$\Delta S = mc_p \ln \left(\frac{T_2}{T_1} \right)$

Constant temperature or isothermal process (Frictionless piston and cylinder)



of

General gas equation $pV = C$

p, V and T relations $p_1 V_1 = p_2 V_2$

Heat supplied = work delivered
 $dQ = dW = p V \ln \left(\frac{V_2}{V_1} \right) = p V \ln \left(\frac{p_1}{p_2} \right)$

Change in internal energy (Change in enthalpy)

$$\Delta U = 0$$

Change in entropy

$$\Delta S = m R \ln \left(\frac{V_2}{V_1} \right) = m R \ln \left(\frac{p_1}{p_2} \right)$$

Reverse Adiabatic Process

{ insulated frictionless piston and cylinder arrangement }

no heat transfer $dQ = 0$

General gas equation $pV^\gamma = C$
p, V and T relations $p_1 V_1^\gamma = p_2 V_2^\gamma$
 $\left(\frac{V_1}{V_2} \right)^{\gamma-1} = \left(\frac{p_2}{p_1} \right)^{\gamma-1}$

Heat delivered

$$Q = 0$$

Work output of the system

$$dW = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

Change in entropy

$$\Delta S = 0$$

Polytropic Process (non-flow polytropic process)

All the processes like isobaric, isochoric, isothermal, adiabatic, etc. may be described by equation

$pV^n = C$ Where 'n' = Index of process.

	If $n = 0$, then	$pV^0 = C$	$p = C$	... Isobaric
If	$n = 1$, then	$pV^1 = C$	$pV = C$	... Isothermal
If	$n = \infty$, then	$pV^\infty = C$	$V = C$	... Isochoric

If $n = \gamma$, then $pV^\gamma = C$ $V = C \dots$ Isentropic

replace γ by n .

p, V and T relations

$$p_1 V_1^n = p_2 V_2^n$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Heat transfer

$$dQ = dU + dW$$

Change in internal energy (Change in enthalpy)

$$\Delta U = m c_v (T_2 - T_1)$$

Work output of the system

$$dW = \frac{p_1 V_1 - p_2 V_2}{n-1}$$

Zeroth law of thermodynamics' states that If two bodies A and B are in thermal equilibrium with a third system C, then the bodies A and B are in thermal equilibrium with each other.

$$a = b \text{ and } b = c, \text{ then } a = c.$$

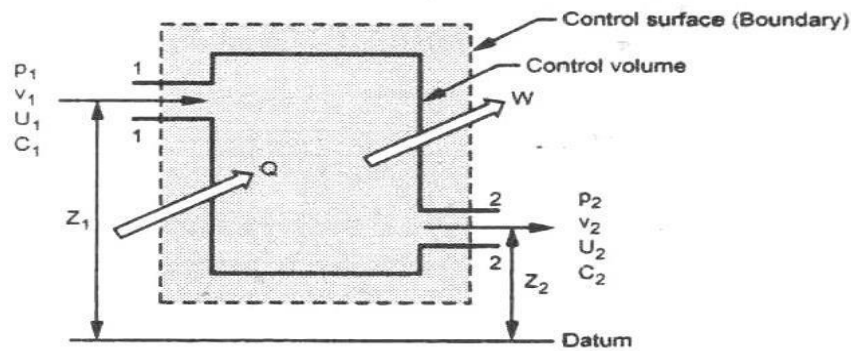
Steady Flow Energy Equation [SFEE]

Consider a steady flow open system as shown

Assumptions :

The following assumptions are made in the system analysis:

- (i) The mass flow through the system remains constant.
- (ii) Fluid is uniform in composition.
- (iii) The only interaction between the system and surroundings are work and heat.
- (iv) The state of fluid at any point remains constant with time.
- (v) In the analysis only potential, kinetic and flow energies are considered.



where, Q = Heat supplied (or entering the boundary) per kg of fluid,
 W = Work done $\frac{kJ}{kg}$

C = Velocity of fluid,

Z = Height above datum,

p = Pressure of the fluid,

u = Internal energy per kg of fluid, and

pv = Energy required for 1 kg of fluid.

This equation is applicable to any medium in

A_1 and A_2 be the cross-

sectional area at inlet and outlet p_1 and p_2 be the pressure at

inlet and outlet.

v_1 and v_2 be the specific volume at inlet and outlet.

U_1 and U_2 be the internal energy at inlet and outlet.

C_1 and C_2 be the velocity for fluid at inlet and outlet.

If Q is the amount of heat interaction W is the work output

Total energy entering the system = P.E + K.E + I.E + F.E + Heat energy

$$= gz_1 + \frac{C_1^2}{2} + u_1 + p_1 v_1 + Q$$

Total energy leaving the system = P.E + K.E + I.E + F.E + Work

$$= gz_2 + \frac{C_2^2}{2} + u_2 + p_2 v_2 + W$$

According to first law of thermodynamics,

$$\text{Total energy entering the system} = \text{Total energy leaving the system}$$

$$gz_1 + \frac{c_1^2}{2} + u_1 + p_1 v_1 + Q = gz_2 + \frac{c_2^2}{2} + u_2 + p_2 v_2 + W$$

$$\therefore [h = u + pv]$$

$$gz_1 + \frac{c_1^2}{2} + h_1 + Q = gz_2 + \frac{c_2^2}{2} + h_2 + W$$

For energy flow per unit of mass

$$m(gz_1 + \frac{c_1^2}{2} + h_1 + Q) = m(gz_2 + \frac{c_2^2}{2} + h_2 + W)$$

for the unit of kJ/kg

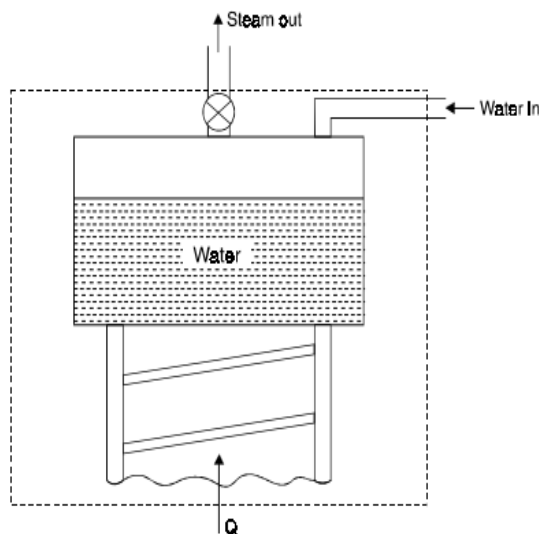
$$m(\frac{gz_1}{1000} + \frac{c_1^2}{2000} + h_1 + Q) = m(\frac{gz_2}{1000} + \frac{c_2^2}{2000} + h_2 + W)$$

Application of Steady Flow Energy Equation

Boiler: (Steam generator)

Internal energy (U) exists and flow energy exists due to movement of water. the P.E (gz) and K.E ($\frac{c^2}{2}$) is negligible

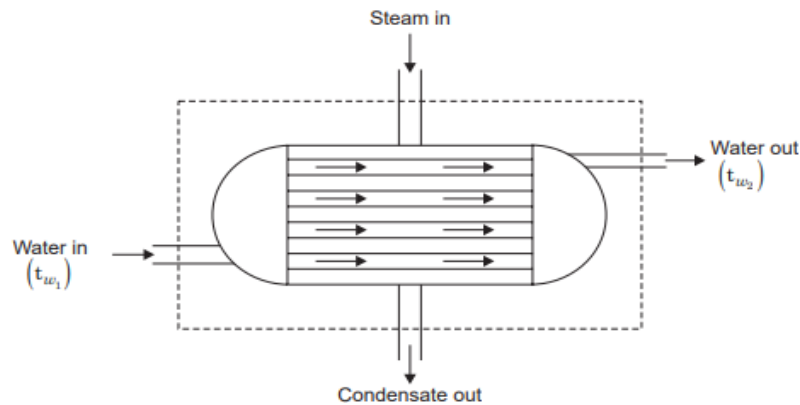
$$\therefore Q = (h_2 - h_1) \text{ kJ}$$



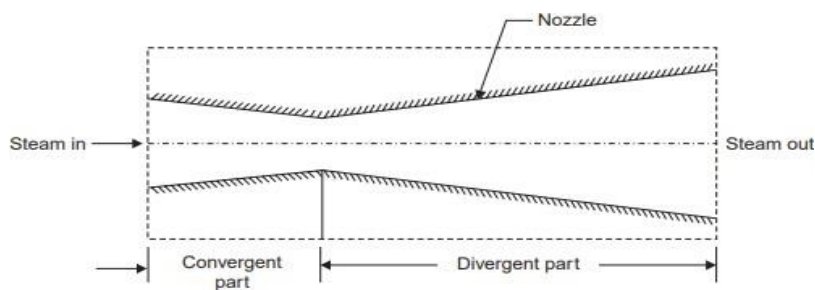
Condenser: Condenser is to transfer the heat from steam to coolant.

There is no work done, change in P.E (gz) and K.E ($\frac{c^2}{2}$) is negligible

$$Q=(h_2-h_1)kJ$$



Nozzle :Increases the velocity of kinetic energy of the working substance at the constant pressure drop.



Now work done by the system ($W=0$)

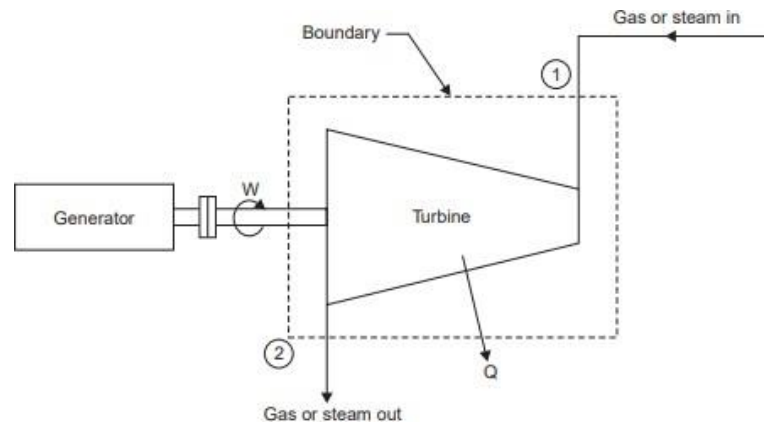
There is no heat transfer taking place ($Q=0$) There is no potential energy ($z_1 = z_2$)

$$\frac{c_1^2}{2} + h_1 = \frac{c_2^2}{2} + h_2$$

Turbine: converts the P.E. into mechanical work.

$$z_1 = z_2, C_1 = C_2$$

Turbine is fully insulated. Therefore, there is no heat transfer



$$Q=0$$

Aircompressor:

- CentrifugalCompressor

No heat transfer

$$\text{change in P.E (gz) and K.E } \left(\frac{c^2}{2} \right)$$

$$W = h_2 - h_1$$

- ReciprocatingCompressor

Applying energy equation to the system,

$$\Delta \text{P.E} = 0 \text{ and } \Delta \text{K.E} = 0$$

since these changes are negligible compared with other energies. $\therefore$

$$h_1 - Q = h_2 - W \quad (-W \text{ is work done on the system})$$

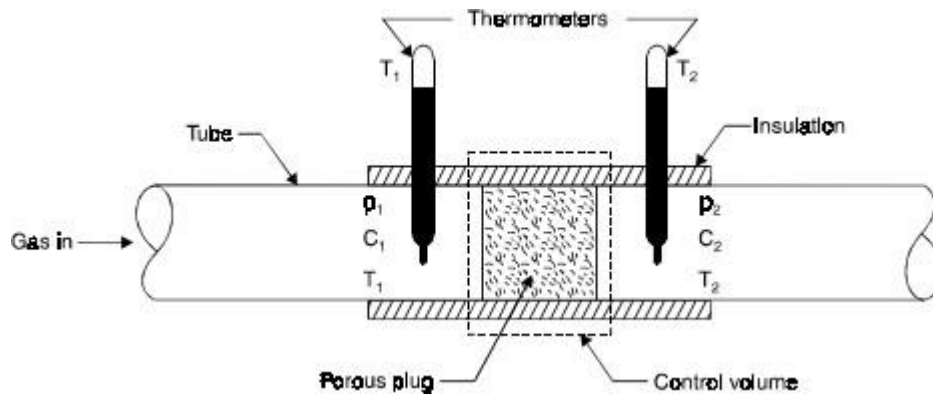
State the limitations of first law of thermodynamics?

1. First Law places no restriction on the direction of a process.
2. It does not ensure whether the process is feasible or not.
3. This law does not differentiate heat and work. It is concerned with the quantity of energy and the transformation of energy from one form to another with no regard to its quality. THROTTLING PROCESS AND JOULE-THROTTLING PROCESS AND JOULE-THOMPSON POROUS PLUG EXPERIMENT

Throttling process involves the passage of a higher pressure fluid through a narrow constriction. The effect is the reduction in pressure and increase in volume. This process is adiabatic as no heat flows from and to the

system, but it is not reversible. It is not an isentropic process. The entropy of the fluid actually increases.

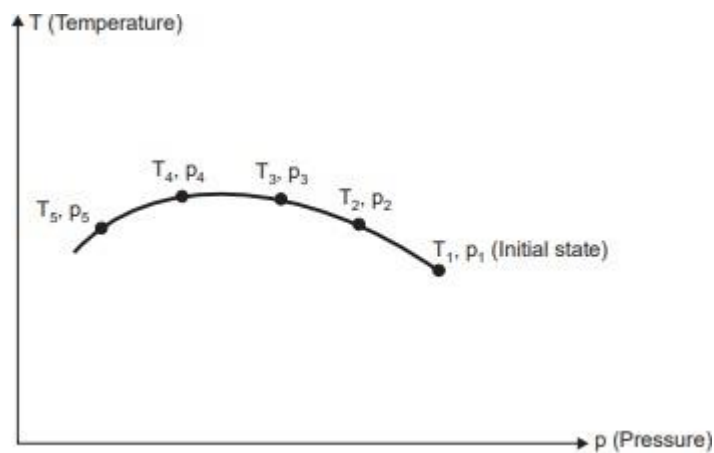
Such a process occurs in a flow through a porous plug, a partially closed valve and a very narrow orifice.



In this system,

$$Q=0 \text{ (System is isolated)}$$

$$W=0 \text{ (There is no work interaction)}$$



$$\Delta PE=0 \text{ (Inlet and outlet are at the same level)}$$

$$\Delta KE=0 \text{ (Kinetic energy does not change significantly)}$$

Energy equation to the system

$$h_1 = h_2$$

This shows that enthalpy remains constant during an adiabatic throttling process.

The throttling process is commonly used for the following purposes: (i) For determining the condition of steam (dryness fraction).

(ii) For controlling the speed of the turbine.

(i) Used in refrigeration plant for reducing the pressure of the refrigerant before entry into the evaporator

slope is called Joule-Thomson coefficient, μ and is given by

$$\mu = \left(\frac{\partial T}{\partial p} \right)_h$$

= 0 for ideal gas.

Example : In a gas turbine unit, the gases flow through the turbine is 15 kg/s and the power developed by the turbine is 12000 kW. The enthalpies of gases at the inlet and outlet are 1260 kJ/kg and 400 kJ/kg respectively, and the velocity of gases at the inlet and outlet are 50 m/s and 110 m/s respectively.

Calculate :

- The rate at which heat is rejected to the turbine, and
- The area of the inlet pipe given that the specific volume of the gases at the inlet is 0.45 m³/kg.

Solution.

Rate of flow of gases, $m = 15$ kg/s

Volume of gases at the inlet, $v = 0.45$ m³/kg

Power developed by the turbine, $P = 12000$ kW

$$\therefore \text{Work done, } W = \frac{12000}{15} = 800 \text{ kJ/kg}$$

Enthalpy of gases at the inlet, $h_1 = 1260$ kJ/kg

Enthalpy of gases at the outlet, $h_2 = 400$ kJ/kg

Velocity of gases at the inlet, $C_1 = 50$ m/s

Velocity of gases at the outlet, $C_2 = 110$ m/s.

Heat rejected, Q :

Using the flow equation,

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W$$

$$1260 + \frac{50^2}{2 \times 1000} + Q = 400 + \frac{110^2}{2 \times 1000} + 800$$

$$1260 + 1.25 + Q = 400 + 6.05 + 800$$

$$\therefore Q = -55.2 \text{ kJ/kg}$$

i.e., Heat rejected = +55.2 kJ/kg = 55.2 × 15 kJ/s = 828 kW. (Ans.)

(ii) Inlet area, A :

Using the relation,

$$m = \frac{CA}{v} = mv \cdot \frac{A}{C}$$

$$= \frac{0.45 \times 50}{50} \cdot 0.135^2$$

Example: 12 kg of air per minute is delivered by a centrifugal air

compressor. The inlet and outlet conditions of air are $C_1 = 12$ m/s, $p_1 = 1$ bar, $v_1 = 0.5$ m³/kg and $C_2 = 90$ m/s, $p_2 = 8$ bar, $v_2 = 0.14$ m³/kg. The

increase in enthalpy of air passing through the compressor is 150 kJ/kg and heat loss to the surroundings is 700 kJ/min.

Find : (i) Motor power required to drive the compressor;

(ii) Ratio of inlet to outlet pipe diameter.

Assume that inlet and discharge lines are at the same level. Solution.

Quantity of air delivered by the compressor, $m = 12 = 0.2 \frac{\text{kg}}{\text{s}}$

Condition of air at the inlet 1:

Velocity, $C_1 = 12 \text{ m/s}$

Pressure, $p_1 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$ Specific

volume, $v_1 = 0.5 \text{ m}^3/\text{kg}$

Condition of air at the outlet 2:

Velocity, $C_2 = 90 \text{ m/s}$

Pressure, $p_2 = 8 \text{ bar} = 8 \times 10^5 \text{ N/m}^2$ Specific

volume, $v_2 = 0.14 \text{ m}^3/\text{kg}$

Increase in enthalpy of air passing through the compressor, $(h_2 - h_1) = 150 \text{ kJ/kg}$

Heat lost to the surroundings, $Q = -700 \text{ kJ/min} = -11.67 \text{ kJ/s}$.

(i) Motor power required to drive the compressor:

Applying energy equation to the system,

$$m(gz_1 + \frac{C_1^2}{2} + h_1) + Q = m(gz_2 + \frac{C_2^2}{2} + h_2) + W$$

For compressor $z_1 = z_2$

$$W = m((h_1 - h_2) + \frac{1}{2 \times 1000} (C_2^2 - C_1^2)) + Q$$

$$= -42.46 \text{ kJ/s} = 42.46 \text{ kW}$$

(ii) Ratio of inlet to outlet pipe diameter, $\frac{d_1}{d_2}$

The mass of air passing through the compressor is given by

$$m = \frac{A_1 C_1}{V_1} = \frac{A_2 C_2}{V_2}$$

$$\frac{A_1}{A_2} = \frac{C_2}{C_1} \times \frac{v_1}{v_2} = \frac{90}{12} \times \frac{0.5}{0.14} = 26.78$$

$$\left(\frac{d_1}{d_2}\right)^2 = 26.78$$

$$\frac{d_1}{d_2} = 5.175$$

A turbo compressor delivers 2.33 m³/s at 0.276 MPa, 43°C which is heated at this pressure to 430°C and finally expanded in a turbine which delivers 1860 kW. During the expansion, there is a heat transfer of 0.09 MJ/s to the surroundings. Calculate the turbine exhaust temperature if changes in kinetic and potential energy are negligible.

$$t = 93^\circ\text{C} \quad V = 2.33 \text{ m}^3/\text{s}; \quad p = 0.276 \text{ MPa}; t = 930^\circ\text{C}$$

$$\frac{dW}{dt} = 1860 \text{ kW} \quad \frac{dQ}{dt} = -0.09 \times 1000 \text{ KJ/s} = -90 \text{ kW}$$

Constant pressure or isobaric process General gas equation $pV = C$ p, V and T relations $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ Heat supplied to the system $= mc_p(T_2 - T_1)$ Work done $dW = p(V_2 - V_1)$ Change in internal energy (Change in enthalpy) $dh = dQ = mc_p(T_2 - T_1)$ Change in entropy $\Delta S = mc_p \ln\left(\frac{T_2}{T_1}\right)$	Constant temperature or isothermal process (Frictionless piston and cylinder) General gas equation $pV = C$ p, V and T relations $p_1V_1 = p_2V_2$ Heat supplied = work delivered $dQ = dW = pV \ln\left(\frac{V_2}{V_1}\right)$ $= pV \ln\left(\frac{p_1}{p_2}\right)$ Change in internal energy (Change in enthalpy) $\Delta U = 0$ Change in entropy $\Delta S = mR \ln\left(\frac{V_2}{V_1}\right) = mR \ln\left(\frac{p_1}{p_2}\right)$
Reverse Adiabatic Process {insulated frictionless piston and cylinder arrangement} no heat transfer $dQ = 0$ General gas equation $pV^\gamma = C$ p, V and T relations $pV^\gamma = C$ $p_2V_2^\gamma = p_1V_1^\gamma$ $\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{\gamma-1}$	Polytropic Process (non-flow polytropic process) All the processes like isobaric, isochoric, isothermal, adiabatic, etc. may be described by equation $pV^n = C$ Where 'n' = Index of process. If $n=0$, then

Heat delivered $Q = 0$ Work output of the system $dW = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$	$pV^0 = C$ $p = C$...Isobaric If $n=1$, then $pV^1 = C$ $pV = C$...Isothermal If $n=\infty$, then $pV^\infty = C$ $V = C$...Isochoric If $n=\gamma$, then $pV^\gamma = C$ $V = C$...Isentropic
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p, V and T relations

$$p_1 V_1^n = p_2 V_2^n$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{n-1}$$

Heat transfer

$$dQ = \Delta U + dw$$

Change in internal energy (Change in enthalpy)

$$\Delta U = m c_v (T_2 - T_1)$$

Work output of the system

$$dw = \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

Change in entropy

$$\Delta S = 0$$

Zeroth law of thermodynamics' states that If two bodies A and B are in thermal equilibrium with a third system C, then the bodies A and B are in thermal equilibrium with each other.

$$a = b \text{ and } b = c, \text{ then } a = c.$$

Steady Flow Energy Equation [SFEE]

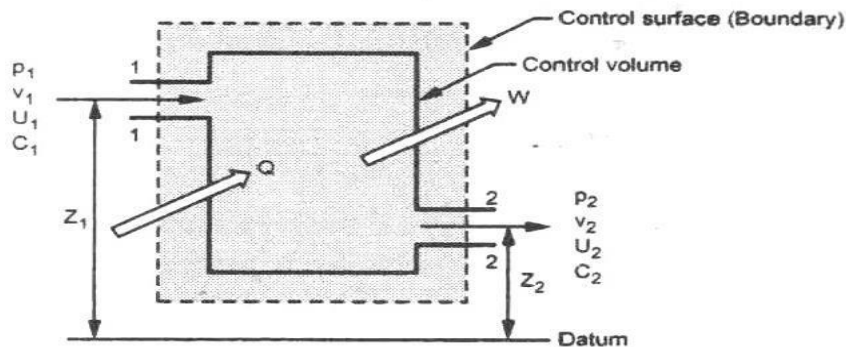
Consider a steady flow open system as shown.

Assumptions :

The following assumptions are made in the system analysis:

- (i) The mass flow through the system remains constant.
- (ii) Fluid is uniform in composition.

- (iii) The only interaction between the system and surroundings are work and heat.
- (iv) The state of fluid at any point remains constant with time.
- (v) In the analysis only potential, kinetic and flow energies are considered.



where, Q = Heat supplied (or entering the boundary) per kg of fluid,

W = Work done $\frac{kJ}{kg}$

C = Velocity of fluid,

Z = Height above datum,

p = Pressure of the fluid,

u = Internal energy per kg of fluid, and

pv = Energy required for 1 kg of fluid.

This equation is applicable to any medium in

A_1 and A_2 be the cross-

sectional area at inlet and outlet p_1 and p_2 be the pressure at

inlet and outlet.

v_1 and v_2 be the specific volume at inlet and outlet.

U_1 and U_2 be the internal energy at inlet and outlet.

C_1 and C_2 be the velocity for fluid at inlet and outlet.

If Q is the amount of heat interaction W is the work output

Total energy entering the system = P.E + K.E + I.E + F.E + Heat energy

$$= gz_1 + \frac{C_1^2}{2} + u_1 + p_1 v_1 + Q$$

Total energy leaving the system = P.E + K.E + I.E + F.E + Work

$$=gz_2 + \frac{c_2^2}{2} + u_2 + p_2 v_2 + W$$

According to first law of thermodynamics,

Total energy [entering the system] = Total energy [leaving the system]

$$gz_1 + \frac{c_1^2}{2} + u_1 + p_1 v_1 + Q = gz_2 + \frac{c_2^2}{2} + u_2 + p_2 v_2 + W$$

$$\therefore [h = u + pv]$$

$$gz_1 + \frac{c_1^2}{2} + h_1 + Q = gz_2 + \frac{c_2^2}{2} + h_2 + W$$

For energy flow per unit of mass

$$m(gz_1 + \frac{c_1^2}{2} + h_1 + Q) = m(gz_2 + \frac{c_2^2}{2} + h_2 + W)$$

for the unit of kJ/kg

$$m(\frac{gz_1}{1000} + \frac{c_1^2}{2000} + h_1 + Q) = m(\frac{gz_2}{1000} + \frac{c_2^2}{2000} + h_2 + W)$$

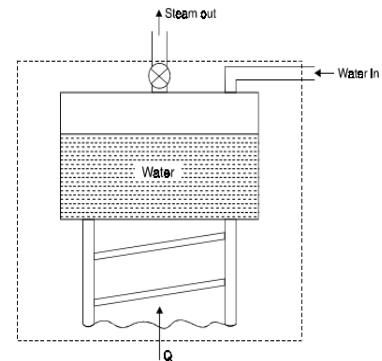
Application of Steady Flow Energy Equation

Boiler: (Steam generator)

Internal energy (U) exists and flow energy exists due to movement of water.

the P.E (gz) and K.E ($\frac{c^2}{2}$) is negligible

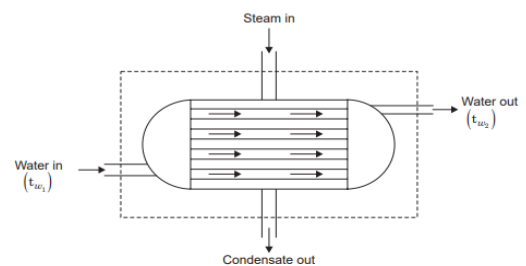
$$\therefore Q = (h_2 - h_1) \text{ kJ}$$



Condenser: Condenser is to transfer the heat from steam to coolant.

There is no work done, change in P.E (gz) and K.E ($\frac{c^2}{2}$) is negligible

$$Q = (h_2 - h_1) \text{ kJ}$$



Nozzle: Increases the velocity of kinetic energy of the working substance at the constant pressure drop.

No work done by the system ($W=0$)

There is no heat transfer taking place ($Q=0$)

There is no potential energy ($z_1=z_2$)

$$\frac{c_1^2}{2} + h_1 = \frac{c_2^2}{2} + h_2$$

Turbine: converts the P.E into mechanical work.

$$z_1=z_2, C_1=C_2$$

Turbine is fully insulated. Therefore, there is no heat transfer

$$Q=0$$

Air compressor:

- Centrifugal Compressor

No heat transfer

change in P.E (gz) and K.E ($\frac{1}{2}c^2$)

$$W = h_2 - h_1$$

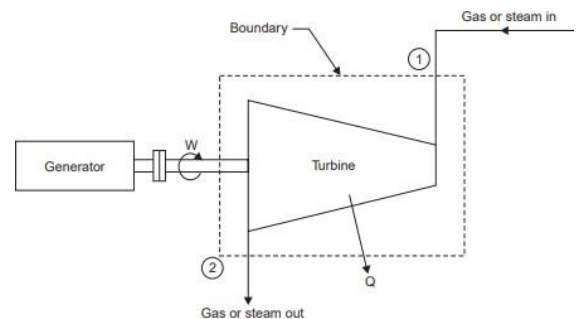
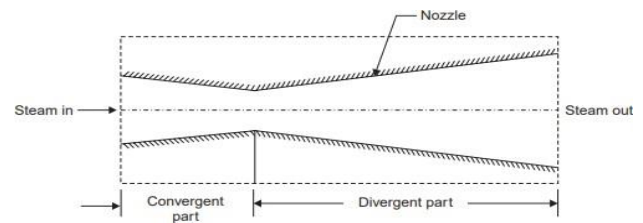
- Reciprocating Compressor

Applying energy equation to the system,

$$\Delta P.E = 0 \text{ and } \Delta K.E = 0$$

since these changes are negligible compared with other energies. $\therefore$

$$h_1 - Q = h_2 - W \quad (-W \text{ is work done on the system})$$



State the limitations of first law of thermodynamics?

1. First Law places no restriction on the direction of a process.
2. It does not ensure whether the process is feasible or not.
3. This law does not differentiate heat and work. It is concerned with the quantity of energy and the transformation of energy from one form to another with no regard to its quality.

Example 1: A fluid system, contained in a piston and cylinder machine, passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is -340 kJ . The system completes 200 cycles per min.

Complete the following table showing the method for each item, and compute the net rate of work output in kW.

Process	Q(kJ/min)	W(kJ/min)	$\Delta E(\text{kJ/min})$
1—2	0	4340	—
2—3	42000	0	—
3—4	-4200	—	-73200
4—1	—	—	—

Solution.

Sum of all heat transferred during the cycle = -340 kJ .

Number of cycles completed by the system = 200 cycles/min.

$$\text{Total heat energy} = -340 \times 200 = -68000\text{ kJ/min}$$

Process 1—2: $Q = \Delta E + W$ $= \Delta E + 4340$ $\therefore \Delta E = -4340\text{ kJ/min.}$	Process 2—3: $Q = \Delta E + W$ $42000 = \Delta E + 0$ $\Delta E = 42000\text{ kJ/min.}$	Process 3—4: $Q = \Delta E + W$ $-4200 = -73200 + W$ $\therefore W = 69000\text{ kJ/min.}$
--	--	--

Process 4—1:

$$\Sigma Q_{\text{cycle}} = -340\text{ kJ}$$

$$Q_{1-2} = Q_{2-3} + Q_{3-4} + Q_{4-1} = -68000\text{ kJ/min}$$

$$0 + 42000 + (-4200) + Q_{4-1} = -68000 \quad Q_{4-1} = -105800\text{ kJ/min.}$$

Total internal energy ΔU or ΔE

$\oint dE = 0$, since cyclic integral of any property is zero.

$$\Delta E_{1-2} + \Delta E_{2-3} + \Delta E_{3-4} + \Delta E_{4-1} = 0$$

$$-4340 + 42000 + (-73200) + \Delta E_{4-1} = 0$$

$$\therefore \Delta E_{4-1} = 35540 \text{ kJ/min.}$$

$$\begin{aligned} \therefore \text{Work done by system } W_{4-1} &= Q_{4-1} - \Delta E_{4-1} \\ &= -105800 - 35540 = -141340 \text{ kJ/min} \end{aligned}$$

Example 2: System of volume V contains a mass m of gas at pressure p and temperature T . The macroscopic properties of the system obey the following relationship:

$$\left(p \pm \frac{a}{V^2}\right)(V - b) = mRT$$

Where a, b , and R are constants.

Obtain an expression for the displacement work done by the system during a constant-temperature expansion from volume V_1 to volume V_2 .

Calculate the work done by a system which contains 10 kg of this gas expanding from 1 m^3 to 10 m^3 at a temperature of 293 K. Use the values $a = 15.7 \times 10 \text{ Nm}^4$, $b = 1.07 \times 10^{-2} \text{ m}^3$, and $R = 0.278 \text{ kJ/kg-K}$.

$$\left(p \pm \frac{a}{V^2}\right)(V - b) = k(mRT)$$

$$\left(p \pm \frac{a}{V^2}\right) = \frac{k}{(V - b)}; p = \frac{k}{(V - b)} - \frac{a}{V^2}$$

$$\left(p_1 + \frac{a}{V_1^2}\right)(V_1 - b) = \left(p_2 + \frac{a}{V_2^2}\right)(V_2 - b) = k(mRT)$$

$$\begin{aligned} W &= \int_1^2 p dV \\ &= \int_1^2 \left(\frac{k}{(V - b)} - \frac{a}{V^2} \right) dV \\ &= \left[k \ln(V - b) + \frac{a}{V} \right] \\ &= k \ln \left(\frac{V_2 - b}{V_1 - b} \right) + a \left(\frac{1}{V_2} - \frac{1}{V_1} \right) \end{aligned}$$

Given $m = 10 \text{ kg}$; $T = 293 \text{ K}$; $R = 0.278 \text{ kJ/kg.K}$

$$\therefore \text{Constant } k = 10 \times 293 \times 0.278 \text{ kJ} = 814.54 \text{ kJ}$$

$$a = 15.7 \times 10 \text{ Nm}^4; b = 1.07 \times 10^{-2} \text{ m}^3$$

$$\Rightarrow V_2 = 10 \text{ m}^3, V_1 = 1 \text{ m}^3$$

$$= 814.54 \ln \left(\frac{10 - 1.07 \times 10^{-2}}{1 - 1.07 \times 10^{-2}} \right) + a \left(\frac{1}{10} - \frac{1}{1} \right)$$

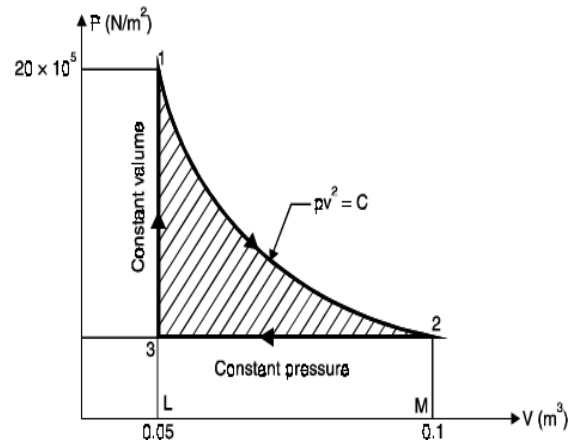
$$= 1742.14 \text{ kJ}$$

Example 3: A cylinder contains 1 kg of a certain fluid at an initial pressure of 20 bar. The fluid is allowed to expand reversibly behind a piston according to a law

$$pV^2 = \text{constant}$$

until the volume is doubled. The fluid is then cooled reversibly at constant pressure until the piston regains its original position; heat is then supplied reversibly with the piston firmly locked in position until the pressure rises to the original value of 20 bar. Calculate the network done by the fluid, for an initial volume of 0.05 m^3 .

Mass of fluid, $m = 1 \text{ kg}$
 $p_1 = 20 \text{ bar} = 20 \times 10^5 \text{ N/m}^2$
 $V_1 = 0.05 \text{ m}^3 \quad v_2 = 2V_1$



Considering the process 1

$$p_1 V_1^2 = p_2 V_2^2$$

$$p_2 = p_1 \frac{V_1^2}{V_2^2} = p_1 \frac{V_1^2}{(2V_1)^2}$$

$$= 20 \times \frac{1}{4} = 5 \text{ bar}$$

Work done by the fluid from 1 to 2 $W =$

$$\int_1^2 p dV$$

$$W_{1-2} = \int_{V_1}^{V_2} \frac{C}{V^2} dV,$$

where $C = p_1 V_1^2 = 20 \times 0.05^2 \text{ bar m}^6$

$$W_{1-2} = 10^5 \times 20 \times 0.0025 \left[\frac{1}{V} \right]_{0.05}^{0.1}$$

$$= 50000 \text{ Nm}$$

Work done on fluid from 2 to 3 (under const. Pr.)

$$p_2 (V_2 - V_3) = 10^5 \times 5 \times (0.1 - 0.05) = 25000 \text{ Nm}$$

Work done during the process 3-1 (Volume remains constant)

$$W = 0,$$

∴ Network done by the fluid

Process 1-2, 2-3, 3-1

$$= 50000 - 25000 + 0$$

$$= 25000 \text{ Nm. (Ans.)}$$

Example. The following equation gives the internal energy of a certain substance

$$u = 3.64pv + 90$$

where u is kJ/kg, p is kPa and v is m³/kg.

*****Example 4:** A system composed of 3.5 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.25 m³ to a final pressure 100 kPa in a process in which pressure and volume are related by

$$pv^{1.25} = \text{constant.}$$

(i) If the expansion is quasi-static, $u = 3.64 pV + 90$

find $Q, \Delta U$ and W for the process.

(ii) In another process, the same system expands according to the same pressure-volume relationship as in part (i) and from the same initial state to the same final state as in part (i) but the heat transfer in this case is 32 kJ.

Find the work transfer for this process.

(iii) Explain the difference in work transfer in parts (i) and (ii).

Solution.

Internal energy equation: $u = 3.64pv + 90$ Initial volume, $V_1 = 0.25 \text{ m}^3$

Initial pressure, $p_1 = 500 \text{ kPa}$

Final pressure, $p_2 = 100 \text{ kPa}$

Process : $pv^{1.25} = \text{constant.}$

$$p_1 V_1^{1.25} = p_2 V_2^{1.25}$$

$$V_2 = V_1 \left(\frac{p_1}{p_2} \right)^{\frac{1}{1.25}} = 0.25 \left(\frac{500}{100} \right)^{\frac{1}{1.25}}$$

$$= 0.906 \text{ m}^3$$

Now, $u = 3.64pv + 90$

$$\Delta u = u_2 - u_1$$

$$= 3.64 (p_2 V_2 - p_1 V_1)$$

$$\therefore \Delta U = 3.64(p_2 v_2 - p_1 v_1)$$

$$\begin{aligned}\Delta U &= 3.64(100 \times 10^3 \times 0.906 - 500 \times 10^3 \times 0.25) \text{ J} [1 \text{ Pa} = 1 \text{ N/m}^2] \\ &= 3.64 \times 10^5 (0.906 - 5 \times 0.25) \text{ J} \\ &= -3.64 \times 10^5 \times 0.344 \text{ J} = -125.2 \text{ kJ} \\ \text{i.e., } \Delta U &= -125.2 \text{ kJ. (Ans.)}\end{aligned}$$

For a quasi-static process Work done

W

$$\begin{aligned}W &= \int p dv = \frac{p_1 V_1 - p_2 V_2}{n-1} \\ &= \frac{500 \times 10^3 \times 0.25 - 100 \times 10^3 \times 0.906}{(1.25-1)} = \frac{125 - 90.6}{0.25} \\ &= 137.6 \text{ KJ}\end{aligned}$$

Heat transfer Q

$$\begin{aligned}\therefore Q &= \Delta U + W \\ &= -125.2 + 137.6 = 12.4 \text{ KJ} \\ \text{i.e., } Q &= 12.4 \text{ kJ. (Ans.)}\end{aligned}$$

(ii) At final states Q = 32 kJ

$$\therefore W = Q - \Delta U = 32 - (-125.2) = 157.2 \text{ kJ. (Ans.)}$$

(iii) The work in (ii) is not equal to $\int p dV$ since the process is not quasi-static.

Example 5: The properties of a system, during a reversible constant pressure nonflow process at $p = 1.6 \text{ bar}$, changed from $v_1 = 0.3 \text{ m}^3/\text{kg}$, $T_1 = 20^\circ\text{C}$ to $v_2 = 0.55 \text{ m}^3/\text{kg}$, $T_2 = 260^\circ\text{C}$. The specific heat of the fluid is given by

$$c_p = \left(1.5 + \frac{75}{T+45}\right) \text{ kJ/kg}^\circ\text{C, where } T \text{ is in } ^\circ\text{C}.$$

Determine:

(i) Heat added/kg; (ii) Work done/kg;
(iii) Change in internal energy/kg; (iv) Change in enthalpy/kg.

Solution.

Initial volume, $v_1 = 0.3 \text{ m}^3/\text{kg}$

Initial temperature, $T_1 = 20^\circ\text{C}$

Final volume, $v_2 = 0.55 \text{ m}^3/\text{kg}$

Final temperature, $T_2 = 260^\circ\text{C}$

Constant pressure, $p = 1.6 \text{ bar} = 1.6 \times 10^5$

Specific heat at constant pressure, $c_p = (1.5 + \frac{75}{T+45})$

$\text{kJ/kg}^\circ\text{C}$

(i) The heat added per kg of fluid is given by

$$Q = \int_{T_1}^{T_2} c_p dT = \int_{20}^{260} (1.5 + \frac{75}{T+45}) dT$$

$$= [1.5T + 75 \log(T+45)]_{20}^{260}$$

Heat added = 475.94 kJ/kg .

(ii) The work done per kg of fluid is given by

$$W = \int_{v_1}^{v_2} p dv = p(v_2 - v_1)$$
$$= 1.6 \times 10^5 (0.55 - 0.3) \text{ N-m}$$
$$= 40 \times 10^3 \text{ J} = 40 \text{ kJ}$$

$\therefore$ Work done = 40 kJ/kg . (Ans.)

(iii) Change in internal energy,

$$\Delta u = Q - W = 475.94 - 40$$
$$= 435.94 \text{ kJ/kg. (Ans.)}$$

(iv) Change in enthalpy, (for non-flow process)

$$\Delta h = Q = 475.94 \text{ kJ/kg. (Ans.)}$$

Example 6: 90 kJ of heat are supplied to a system at a constant volume.

The system rejects 95 kJ of heat at constant pressure and 18 kJ of work is done on it. The system is brought to original state by adiabatic process.

Determine:

(i) The adiabatic work;

(ii) The values of internal energy at all end states if initial value is 105 kJ .

Solution.

Heat supplied at constant volume = 90 kJ

Heat rejected at constant pressure = -95 kJ

Workdoneonthesystem= -18 kJ

Initialvalueofinternalenergy, $U_1=105\text{ kJ}$

Processl-m(constantvolume)	Processm-n(constantpressure)
workdone=0 $Q_{l-m}=90=U_m-U_1$ $\therefore U_m=U_1+90=105+90$ $=195\text{ kJ}$	$Q_{m-n}=(U_n-U_m)+W_{m-n}$ $95=(U_n-U_m)-18$ $\therefore U_n-U_m=-77\text{ kJ}$ $U_n=195-77=118\text{ kJ}$

$Q_{n-l}=0$ beingadiabaticprocess

$$\therefore \delta Q = 90 - 95 = -5\text{ kJ}$$

$$\delta W = -18 + W_{n-l} = -5$$

$$\therefore W_{n-l} = -5 + 18 = 13\text{ kJ}$$

Hence, $W_{n-l} = 13\text{ kJ}$; $U_1=105\text{ kJ}$; $U_m=195\text{ kJ}$; $U_n=118\text{ kJ}$. (Ans.)

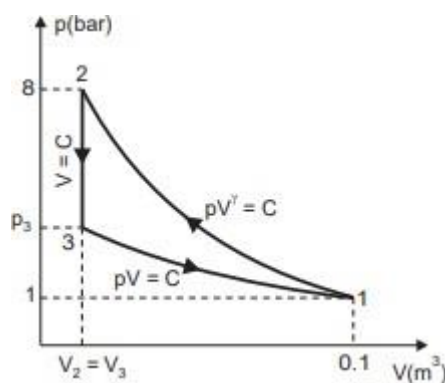
$= 475.94\text{ kJ} \therefore$ Heat added $= 475.94\text{ kJ/kg}$. (Ans.)

Example7 : 0.1 m^3 of an ideal gas at 300 K and 1 bar is compressed adiabatically to 8 bar . It is then cooled at constant volume and further expanded isothermally so as to reach the condition from where it started. Calculate :

- Pressure at the end of constant volume cooling.
- Change in internal energy during constant volume process.
- Network done and heat transferred during the cycle.

Assume $c_p = 14.3\text{ kJ/kg K}$ and $c_v = 10.2\text{ kJ/kg K}$.

Solution. Given : $V_1 = 0.1\text{ m}^3$; $T_1 = 300\text{ K}$; $p_1 = 1\text{ bar}$; $c_p = 14.3\text{ kJ/kg K}$; $c_v = 10.2\text{ kJ/kg K}$.



RefertoFig.

(i) Pressure at the end of constant volume cooling, p_2 :

$$\gamma = \frac{c_p}{c_v} = \frac{14.3}{10.2} = 1.402$$

Characteristic gas constant, R

$$R = c_p - c_v = 14.3 - 10.2 = 4.1 \text{ kJ/kgK}$$

Considering process 1-2,

Rev. adiabatically

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{\gamma-1}$$

$$V_2 = V_1 \times \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma-1}} = 0.1 \times \left(\frac{8}{1}\right)^{\frac{1}{1.402-1}}$$

$$= \mathbf{0.0227 \text{ m}^3}$$

Also,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{8}{1}\right)^{\frac{1.402-1}{1.402}} = 1.815$$

$$T_2 = T_1 \times 1.815 = 300 \times 1.815 = \mathbf{544.5 \text{ K}}$$

Considering process 3-1,

$$p_3 V_3 = p_1 V_1$$

$$\therefore p_3 = \frac{p_1 V_1}{V_3} = \frac{1 \times 0.1}{0.0227} = \mathbf{4.4 \text{ bar. (Ans.)}}$$

(ii) Change in internal energy during constant volume process, $(U_3 - U_2)$

$$\text{Mass of gas, } m = \frac{p_1 V_1}{RT_1} = \frac{(1 \times 10^5) \times 0.1}{4.1 \times 1000 \times 300} = 0.00813 \text{ kg}$$

Change in internal energy during constant volume process 2-3,

$$U_3 - U_2 = mc_v (T_3 - T_2) = 0.00813 \times 10.2 (300 - 544.5)$$

$$(T_3 = T_1)$$

$$= \mathbf{-20.27 \text{ kJ. (Ans.)}} \text{ (—ve sign means decrease in internal energy)}$$

iii) Network done and heat transferred during the cycle:

Example 8: A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m³ to 0.4 MPa, 0.03 m³. Assuming that the pressure and volume are related by $pV^n = \text{constant}$, find the work done by the gas system.

solution: initial pressure $p_1 = 80 \text{ kPa}$

Initial volume $V_1 = 0.1 \text{ m}^3$

Final pressure (p_2) = 0.4 MPa = 400 kPa
Final volume (V_2) = 0.03 m³

As $pV^n = C$

$$p_1 V_1^n = p_2 V_2^n \quad \text{take } \ln$$

g $\log_e$ both side

$$n \ln \left(\frac{V_1}{V_2} \right) = \ln \left(\frac{p_2}{p_1} \right)$$

$$n = \frac{\ln \left(\frac{p_2}{p_1} \right)}{\ln \left(\frac{V_1}{V_2} \right)} = \frac{\ln \left(\frac{400}{80} \right)}{\ln \left(\frac{0.1}{0.03} \right)} = \frac{1.60944}{1.20397} = 1.34$$

$$\begin{aligned} \text{Work done } W &= dW = \frac{p_1 V_1 - p_2 V_2}{n-1} = \frac{80 \times 0.1 - 400 \times 0.03}{1.34 - 1} \\ &= -11.764 \text{ kJ} \end{aligned}$$

Example 9: A mass of air is initially at 260°C and 700 kPa, and occupies 0.028 m³. The air is expanded at constant pressure to 0.084 m³. A polytropic process with $n = 1.5$ is then carried out followed by a constant temperature process which completes a cycle. All the processes are reversible.

i) Sketch the cycle in p - V planes.

ii) Find the heat received and heat rejected in the cycle. Find the efficiency of the cycle.

Sol. : Given : $T_1 = T_3 = 260 + 273 = 533 \text{ K}$,

$$p_1 = p_2 = 700 \text{ kPa} = 700 \text{ kN/m}^2$$

$$V_1 = 0.028 \text{ m}^3, V_2 = 0.084 \text{ m}^3, pV^{1.5} = \text{Constant} \quad (n = 1.5 \text{ given})$$

1-2 = const. pressure ($p = C$)

process 2-3 = polytropic process ($pV^{1.5} = \text{constant}$) process

1-3 = const. temperature ($T = \text{constant}$)

Calculate mass of air

Gas equation, $P_1 V_1 = m R T_1$

$$m = \frac{P_1 V_1}{R T_1} = \frac{700 \times 0.028}{0.287 \times 533} = \mathbf{0.1283 \text{ kg}}$$

Heat supplied on process 1-2 (cont. pressure) $P_1 = P_2$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{700 \times 0.028}{533} = \frac{700 \times 0.084}{T_2}$$

$$T_2 = 1599 \text{ K}$$

Heat transfer

$$Q = m c_p (T_2 - T_1)$$

$$= 0.1283 \times 1.055 \times (1599 - 533)$$

$$\mathbf{Q_{1-2} = 137.13 \text{ kJ}}$$

Heat transfer during process 2-3

$$Q_{2-3} = m c_v (T_3 - T_2) \left(\frac{n-\gamma}{n-1} \right)$$

$$= 0.1283 \times 0.718 \times \left(\frac{1.5-1.4}{1.5-1} \right) \times (533 - 1599)$$

$$= -19.594$$

Heat transfer during process 3-1

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\frac{1.5}{1.5-1}}$$

$$\frac{700}{P_3} = 27$$

$$P_3 = 25.93 \text{ kPa}$$

$$\text{Heat transfer } Q_{3-1} = m R T_1 \ln \frac{P_3}{P_1}$$

$$= 0.1283 \times 0.287 \times 533 \ln \frac{25.93}{700}$$

$$= -64.53 \text{ kJ} \text{ (negative sign indicates heat rejected from system)}$$

$$\text{Heat supplied } Q_S = 137.5 \text{ kJ}$$

$$\text{Heat rejected } Q_R = Q_{R1} + Q_{R2}$$

$$= 19.594 + 64.53$$

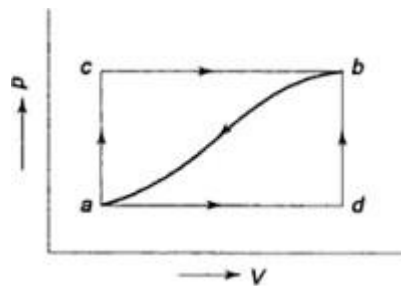
$$= 84.124$$

Efficiency of cycle η

$$\eta = \frac{Q_S - Q_R}{Q_S} = \frac{137.13 - 84.124}{137.13}$$

$$\eta = 38.654\%$$

Example 10A When a system is taken from state *a* to state *b*, in Fig. Ex. along path *acb*, 84 kJ of heat flow into the system, and the system does 32 kJ of work, (a) How much will the heat that flows into the system along path *adb* be, if the work done is 10.5 kJ? (b) When the system is returned from *b* to *a* along the curved path, the work done on the system is 21 kJ. Does the system absorb or liberate heat, and how much of the heat is absorbed or liberated? (c) If $U_a = 0$ and $U_d = 42$ kJ, find the heat absorbed in the processes *ad* and *db*.



$$Q_{acb} = 84 \text{ kJ}, W_{acb} = 32 \text{ kJ}$$

$$Q_{acb} = U_b - U_a + W_{acb}$$

$$U_b - U_a = 84 - 32 = 52 \text{ kJ}$$

$$Q_{adb} = U_b - U_a + W_{adb}$$

$$= 52 + 10.5 = 62.5 \text{ kJ}$$

$$Q_{b-a} = U_a - U_b + W_{b-a}$$

$$= -52 - 21 = -73 \text{ kJ}$$

The system liberates 73 kJ of heat W_{adb}

$$= W_{ad} + W_{db} = W_{ad} = 10.5 \text{ kJ}$$

$$Q_{ad} = U_d - U_a + W_{ad}$$

$$= 42 - 0 + 10.5 = 52.5 \text{ kJ}$$

S.F.E.E.

Steady flow Energy equation

$$Q - W = m[\Delta h + \Delta K.E + \Delta P.E]$$

$$\Delta K.E = \frac{C_1^2 - C_2^2}{2000 \text{ KG}}, \Delta P.E = g \left(\frac{z_1 - z_2}{1000} \right) \frac{\text{kJ}}{\text{KG}}$$

$$\Delta h = (h_2 - h_1) \text{ or } c_p(T_2 - T_1) \text{ or } \Delta u + \Delta pV$$

$$\text{Mass flow rate } \dot{m} = \frac{\text{mass}}{\text{sec}} \text{ kg/s.}$$

Example 1 : 10 kg of fluid per minute goes through a reversible steady flow process. The properties of fluid at the inlet are : $p_1 = 1.5 \text{ bar}$, $\rho_1 = 26 \text{ kg/m}^3$, $C_1 = 110 \text{ m/s}$ and $u_1 = 910 \text{ kJ/kg}$ and at the exit are $p_2 = 5.5 \text{ bar}$, $\rho_2 = 5.5 \text{ kg/m}^3$, $C_2 = 190 \text{ m/s}$ and $u_2 = 710 \text{ kJ/kg}$. During the passage, the fluid rejects 55 kJ/s and rises through 55 metres. Determine :

- (i) The change in enthalpy (Δh);
- (ii) Work done during the process (W).

Solution.

Flow of fluid = 10 kg/min

Properties of fluid at the inlet: Pressure, $p_1 = 1.5 \text{ bar} = 1.5 \times 10^5 \text{ N/m}^2$ Density, $\rho_1 = 26 \text{ kg/m}^3$ Velocity, $C_1 = 110 \text{ m/s}$ Internal energy, $u_1 = 910 \text{ kJ/kg}$	Properties of the fluid at the exit : Pressure, $p_2 = 5.5 \text{ bar} = 5.5 \times 10^5 \text{ N/m}^2$ Density, $\rho_2 = 5.5 \text{ kg/m}^3$ Velocity, $C_2 = 190 \text{ m/s}$
---	---

Heat rejected by the fluid, $Q = 55 \text{ kJ/s}$

Rise is elevation of fluid = 55 m.

(i) The change in enthalpy,

$$\Delta h = \Delta u + \Delta(pv)$$

$$\Delta(pv) = \frac{p_2 v_2 - p_1 v_1}{1} ; p_2 = p_1 \left(v = \frac{1}{\rho} \right)$$

$$\frac{5.5 \times 10^5}{5.5} - \frac{1.5 \times 10^5}{26}$$

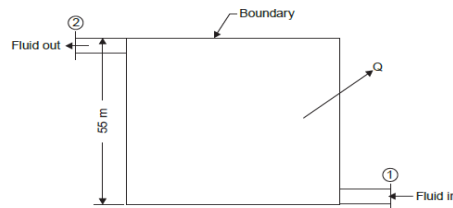
$$= 1 \times 10^5 - 0.0577 \times 10^5$$

$$= 10^5 \times 0.9423 \text{ Nm or J} = 94.23 \text{ kJ}$$

$$\Delta u = u_2 - u_1 = (710 - 910) = -200 \text{ kJ/kg}$$

Substituting the value in eqn. (i), we get

$$\Delta h = -200 + 94.23 = -105.77 \text{ kJ/kg. (Ans.)}$$



Steady flow Energy equation

$$Q - W = m[\Delta h + \Delta K.E + \Delta P.E]$$

$$\Delta K.E = \frac{C_1^2 - C_2^2}{2000 \text{ KG}}, \Delta P.E = g \left(\frac{z_1 - z_2}{1000} \right) \frac{\text{kJ}}{\text{KG}}$$

/M-therm/Th4-4.pm5

$$Q = 55 \text{ kJ/s} = \frac{55}{10} = 330 \text{ kJ/kg}$$

$$\text{Now, } \Delta K.E = \frac{C_1^2 - C_2^2}{2000} = \frac{190^2 - 110^2}{2000} = 12000 \text{ J}$$

$$= 12 \text{ kJ/kg}$$

$$\Delta P.E = (Z_2 - Z_1)g = (55 - 0) \times 9.81 \text{ Nm or J} = 539.5 \text{ J}$$

$$= 0.54 \text{ kJ/kg}$$

Substituting the value in steady flow equation, –

$$330 = 12 + 0.54 - 105.77 + W$$

$$W = -236.77 \text{ kJ/kg.}$$

$$\text{Work done per second} = -236.77 \times \frac{10}{60} = -39.46 \text{ kJ/s}$$

$$= -39.46 \text{ kW. (Ans.)}$$

Example 2: Air at a temperature of 15°C passes through a heat exchanger at a velocity of 30 m/s where its temperature is raised to 800°C . It then enters a turbine with the same velocity of 30 m/s and expands until the temperature falls to 650°C . On leaving the turbine, the air is taken at a velocity of 60 m/s to a nozzle where it expands until the temperature has fallen to 500°C . If the air flow rate is 2 kg/s , calculate (a) the rate of heat transfer to the air in the heat exchanger, (b) the power output from the turbine assuming no heat loss, and (c) the velocity at exit from the nozzle, assuming no heat loss. Take the enthalpy of air as $h = c_p t$, where c_p is the specific heat equal to 1.005 kJ/kg K and t the temperature.

Solution As shown in Fig. writing the S.F.E.E. for the heat exchanger and eliminating the terms not relevant,

Inlet of heat exchanger	$T_1 = 15 + 273 = 288 \text{ K}$	$c_1 = 30 \text{ m/s}$
turbine	$T_2 = 800 + 273 = 1073$	$c_2 = 30 \text{ m/s}$
Nozzle inlet	$T_3 = 650 + 273 = 923 \text{ K}$	$c_3 = 60 \text{ m/s}$
Nozzle outlet	$T_4 = 500 + 273 = 773 \text{ K}$	$c_4 = ?$

air flow rate $w = 2 \text{ kg/s}$, specific heat $c_p = 1.005$

enthalpy of air as $h = c_p t$

To find:

(a) heat transfer to the air in the heat exchanger,

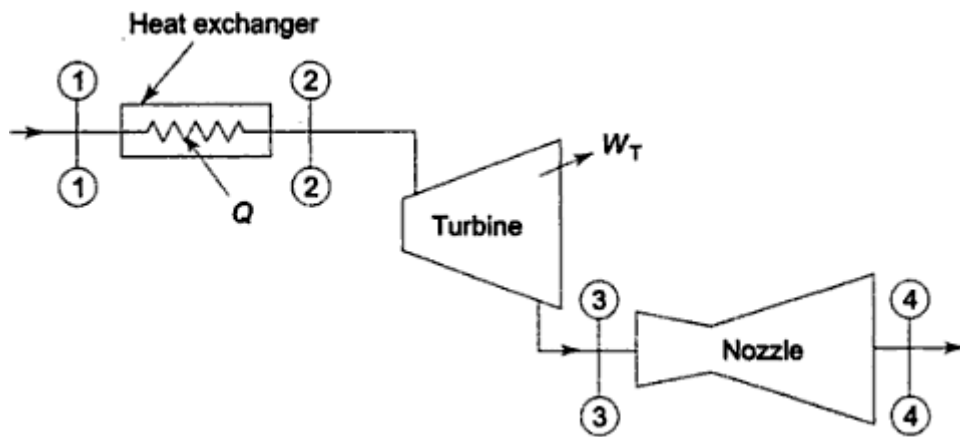
(b) the power output

(c) the velocity at exit from the nozzle

Steady flow Energy equation

$$m \left(gz_1 + \frac{c_1^2}{2} + h_1 \right) + Q = m \left(gz_2 + \frac{c_2^2}{2} + h_2 \right) + W$$

$$Q - W = m [\Delta h + \Delta \text{K.E.} + \Delta \text{P.E.}]$$



$$\Delta K.E = \frac{c_1^2 - c_2^2}{2000} \text{ kJ/KG}, \Delta P.E = g \left(\frac{z_1 - z_2}{1000} \right) \frac{\text{kJ}}{\text{KG}}$$

In heat exchanger

$$W_{1-2} = 0 \quad z_1 = z_2$$

Steady flow energy equation reduces to

$$m \left(\frac{c_1^2}{2} + h_1 \right) + Q = m \left(\frac{c_2^2}{2} + h_2 \right)$$

From given data $c_1 = c_2$ Q

$$= m(h_2 - h_1); m c_p(T_2 - T_1)$$

$$= m c_p(T_2 - T_1) = 2 \times 1.005 \times (1073 - 288)$$

$$Q_{1-2} = 1577.85 \text{ kW}$$

For turbine 2-3

$$Q_{2-3} = 0 \quad z_2 = z_3$$

Steady flow energy equation reduces to

$$m \left(\frac{c_2^2}{2} + h_2 \right) = m \left(\frac{c_3^2}{2} + h_3 \right) + W$$

$$\Delta h = (h_2 - h_3) \text{ or } c_p(T_2 - T_3) \text{ or } \Delta u + \Delta pV$$

$$W = (h_2 - h_3) + \left(\frac{c_2^2}{2000} - \frac{c_3^2}{2000} \right)$$

$$W + c_p(T_2 - T_3) + \left(\frac{c_2^2}{2000} - \frac{c_3^2}{2000} \right)$$

$$W + 1.005(1073 - 923) + \left(\frac{30^2}{2000} - \frac{60^2}{2000} \right)$$

$$W = 2988 \text{ kW}$$

Fornozzle3-4

$$\text{Assume that } Q_{3-4} = 0 \quad z_3 = z_4 (\text{adiabatic nozzle}) \text{ and } W_{3-4} = 0$$

Steady flow Energy equation reduces to

$$c_p(T_3 - T_4) + \frac{c_3^2 - c_4^2}{2000}$$

$$1.005(923 - 773) + \frac{60^2 - c_4^2}{2000}$$

$$c_4 = 552.36 \frac{\text{m}}{\text{s}}$$

Example 3: In a gas turbine unit, the gases flow through the turbine is 15 kg/s and the power developed by the turbine is 12000 kW. The enthalpies of gases at the inlet and outlet are 1260 kJ/kg and 400 kJ/kg respectively, and the velocity of gases at the inlet and outlet are 50 m/s and 110 m/s respectively. Calculate :

(i) The rate at which heat is rejected to the turbine, and

(ii) The area of the inlet pipe given that the specific volume of the gases at the inlet is 0.45 m³/kg.

Solution. Rate of flow of gases, $\dot{m} = 15 \text{ kg/s}$

Volume of gases at the inlet, $v = 0.45 \text{ m}^3/\text{kg}$

Power developed by the turbine, $P = 12000 \text{ kW}$

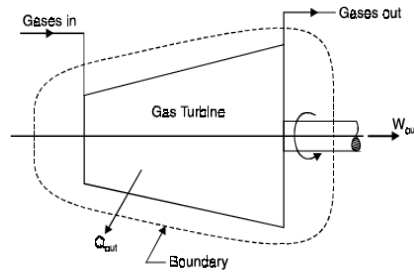
$\therefore$ Work done, $W = \frac{12000}{15} = 800 \text{ kJ/kg}$

Enthalpy of gases at the inlet, $h_1 = 1260 \text{ kJ/kg}$

Enthalpy of gases at the outlet, $h_2 = 400 \text{ kJ/kg}$

Velocity of gases at the inlet, $C_1 = 50 \text{ m/s}$

Velocity of gases at the outlet, $C_2 = 110 \text{ m/s}$.



Steady flow Energy equation

$$Q - W = m[\Delta h + \Delta K.E + \Delta P.E]$$

$$\Delta h = (h_2 - h_1) = (400 - 1260) = -860 \text{ kJ/kg}$$

$$\Delta P.E = 0 \text{ [gas turbine } z_1 = z_2]$$

$$\begin{aligned} \Delta K.E &= \frac{C_2^2 - C_1^2}{2000} \text{ kJ/kg} \\ &= \frac{110^2 - 50^2}{2000} = 4.8 \text{ kJ/kg} \end{aligned}$$

$$Q = 15 (800 - 860 + 4.8) = 828 \text{ kJ/s}$$

Heat rejected = 828 kW. (Ans.)

ii) Inlet area, A:

Using the relation,

$$\dot{m} = \frac{CA\rho}{v}$$

$$A = \frac{\dot{m} v}{C \rho} = \frac{0.45 \times 15}{50} = 0.135 \text{ m}^2$$

A = 0.135 m². (Ans.)

Example 4: Steam at 6.87 bar, 205°C, enters in an insulated nozzle with a velocity of 50 m/s. It leaves at a pressure of 1.37 bar and a velocity of 500 m/s. Determine the final enthalpy of steam.

Solution. Pressure of steam at the entrance, $p_1 = 6.87 \text{ bar}$

The velocity with which steam enters the nozzle, $C_1 = 50 \text{ m/s}$ Pressure of steam at the exit, $p_2 = 1.37 \text{ bar}$

Velocity of steam at the exit, $C_2 = 500 \text{ m/s}$.

To find: final enthalpy of steam.

Steady flow Energy equation

$$Q - W = m[\Delta h + \Delta K.E + \Delta P.E]$$

Considering the insulated nozzle

— there is no work transfer (i.e., $W=0$)

— there is no heat transfer (i.e., $Q=0$).

— the change in potential energy is negligible [i.e. $(Z_2 - Z_1)g = 0$].

Mass flow rate $m=1$

Equation becomes $\Delta h + \Delta K.E = 0$

From steam table corresponding to 6.87 bar, $h_1 = 2850 \text{ kJ/kg}$

$$\Delta h = (h_2 - h_1) = (h_2 - 2850)$$

$$\Delta P.E = 0 \quad [z_1 = z_2]$$

$$\Delta K.E = \frac{C_2^2 - C_1^2}{2000} \text{ kJ/kg}$$

$$= \frac{500^2 - 50^2}{2000}$$

$$(h_2 - h_1) + \frac{C_2^2 - C_1^2}{2000} = 0 \quad (h_2 - 2850) + \frac{500^2 - 50^2}{2000} = 0$$

$$h_2 = 2726.25 \text{ kJ}$$

Hence final enthalpy of steam = 2726.25 kJ. (Ans.)

Example 5: 12 kg of air per minute is delivered by a centrifugal air compressor. The inlet and outlet conditions of air are $C_1 = 12 \text{ m/s}$, $p_1 = 1 \text{ bar}$, $v_1 = 0.5 \text{ m}^3/\text{kg}$ and $C_2 = 90 \text{ m/s}$, $p_2 = 8 \text{ bar}$, $v_2 = 0.14 \text{ m}^3/\text{kg}$. The increase in enthalpy of air passing through the compressor is 150 kJ/kg and heat loss to the surroundings is 700 kJ/min . Find : (i) Motor power required to drive the compressor ;

(ii) Ratio of inlet to outlet pipe diameter.

Assume that inlet and discharge lines are at the same level.

Solution. Quantity of air delivered by the compressor, $m = 12 = 0.2 \text{ kg/s}$.

Conditions of air at the inlet 1:

Velocity, $C_1 = 12 \text{ m/s}$

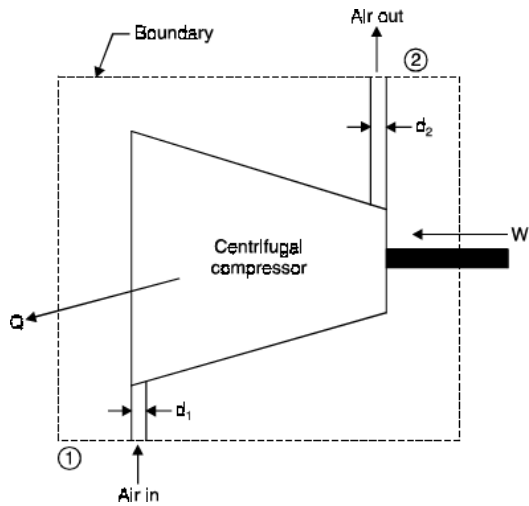
Pressure, $p_1 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$

Specific volume, $v_1 = 0.5 \text{ m}^3/\text{kg}$

Conditions of air at the outlet 2:

Velocity, $C_2 = 90 \text{ m/s}$

Pressure, $p_2 = 8 \text{ bar} = 8 \times 10^5 \text{ N/m}^2$



Specific volume, $v_2 = 0.14 \text{ m}^3/\text{kg}$

Increase in enthalpy of air passing through the compressor, $(h_2 - h_1) = 150 \text{ kJ/kg}$

Heat lost to the surroundings,

$Q = -700 \text{ kJ/min} = -11.67 \text{ kJ/s}$.

i) Motor power required to drive the compressor:

Steady flow Energy equation

$$Q - W = m[\Delta h + \Delta K.E + \Delta P.E]$$

$$\Delta P.E = 0 \text{ [for compressor]}$$

$$Q - W = m[\Delta h + \Delta K.E]$$

$$\Delta h = (h_2 - h_1); (h_2 - h_1) = 150 \text{ kJ/kg}$$

$$\Delta h = 150 \text{ kJ/kg}$$

$$\Delta K.E = \frac{C_2^2 - C_1^2}{2000} \text{ kJ/kg}$$

$$= \frac{12^2 - 90^2}{2000} = 3.978 \text{ kJ/kg}$$

$$-11.67 - W = 0.2[-150 + 3.978]$$

$$W = -42.46 \text{ kJ/s} = -42.46 \text{ kW}$$

Motor power required (or work done on the air) = 42.46 kW. (Ans.)

ii) Ratio of inlet to outlet pipe diameter, $\frac{d_1}{d_2}$

The mass of air passing through the compressor is given by

$$m = \frac{C_1 A_1}{v_1} = \frac{C_2 A_2}{v_2}$$

$$\frac{A_1}{A_2} = \frac{C_2}{C_1} \times \frac{v_1}{v_2} = \frac{90}{12} \times \frac{0.5}{0.14} = 26.78$$

$$\left(\frac{d_1}{d_2}\right)^2 = 26.78$$

Hence ratio of inlet to outlet pipe diameter = 5.175. (Ans.)

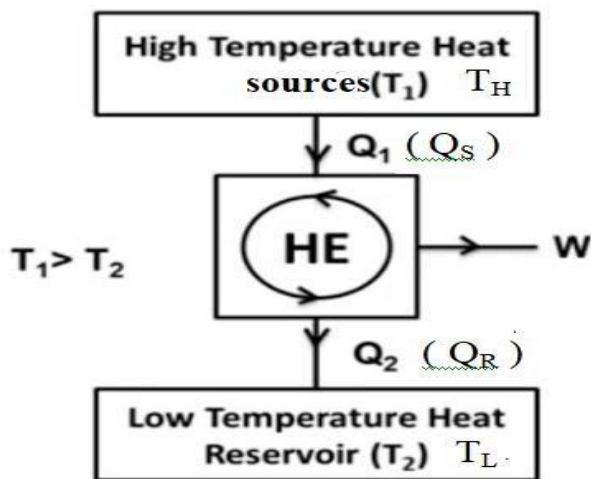
Unit-2

HeatEngine

HeatEngines: Heat engine is a cyclic device, used to convert heat to work.

Heat engine can be characterized by the following points.

1. They receive heat from a high temperature source (solar energy, oil- furnace etc.)
2. They convert part of this heat to work (usually in the form of a rotating shaft)
3. They reject the remaining waste heat to a low temperature sink (the atmosphere, rivers, etc)
4. They operate on a cycle.



Q_1 = amount of heat supplied to steam in boiler from a high-temperature source.

Q_2 = amount of heat rejected from steam in condenser to a low temperature sink.

W = network output of this heat engine.

From first law of thermodynamics,

Heat transfer from sources $Q_1 = W + Q_2$

Work done $W = Q_1 - Q_2$

Thermal efficiency (η) = $\frac{\text{Network output}}{\text{Total heat input}}$

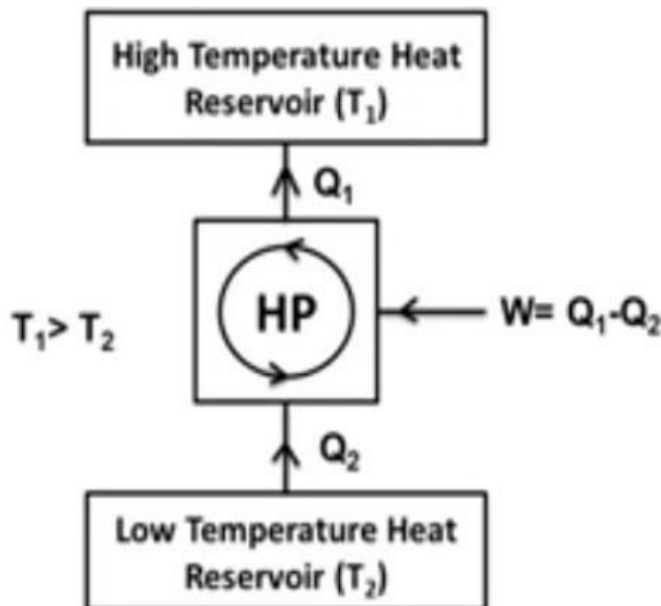
$$\eta_{th} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1}$$

• **Hence, efficiency of heat engine is always less unity.**

Heat Pump

Heat Pumps: Heat pumps are another cyclic devices, used to transfer heat from a low temperature medium to a high temperature medium. The objective of a heat pump is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from a low temperature source, such as cold outside air in winter and supplying this heat to the high temperature medium such as a house.



$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_1}{W}$$

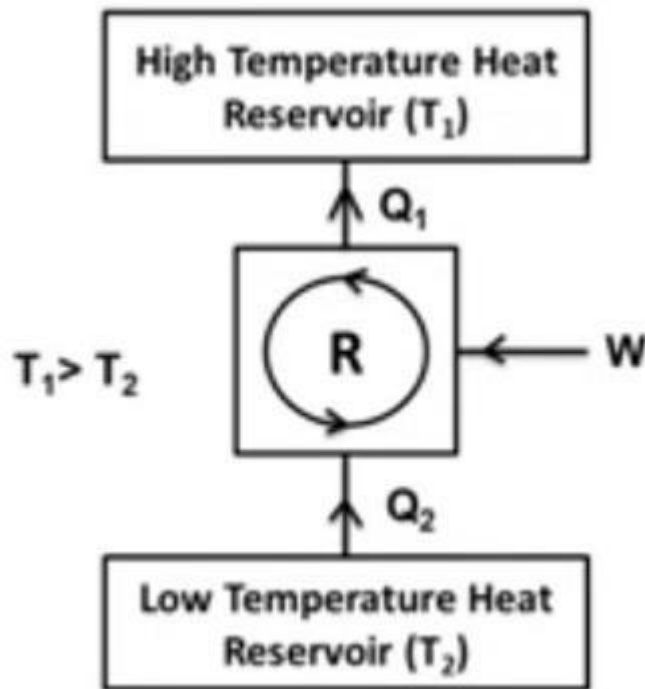
$$COP_{HP} = \frac{Q_1}{Q_1 - Q_2}$$

For reversible heat pump (Carnot) $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$$COP_{HP(rev)} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

Refrigerator

Desired effect of refrigerator is to cool the given space by removing heat



$$\text{COP}_{\text{Ref}} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_2}{W}$$

$$\text{COP}_{\text{Ref}} = \frac{Q_2}{Q_1 - Q_2}$$

For reversible heat pump (Carnot) $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$$\text{COP}_{\text{Ref(rev)}} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

Relation between COP_{HP} and COP_R

$$\text{COP}_{\text{HP}} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$$

$$\text{COP}_{\text{Ref}} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\text{By eqn: (1) - (2) } \text{COP}_{\text{HP}} - \text{COP}_{\text{Ref}} = \frac{Q_1}{Q_1 - Q_2} - \frac{Q_2}{Q_1 - Q_2} = 1$$

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{R}} + 1$$

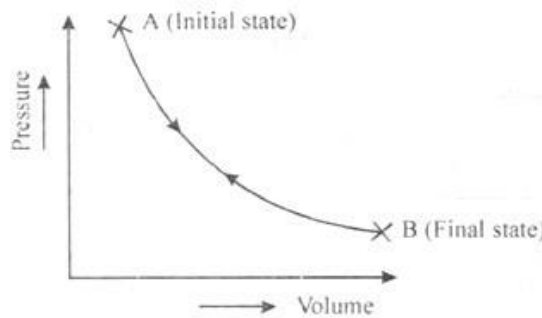
Reversible Process and Irreversible Process

A reversible process is one which is performed in such a way that both system and surroundings may be restored to their initial state, without producing any changes

The assumptions made for reversible process

1. The reversible process has to be a quasi-equilibrium process
2. No friction is involved in the process
3. Heat transfer occurs due to an infinitesimal temperature difference only
4. Unrestrained expansion does not occur.

The reversible process is purely an ideal process and it is an imaginary process which is used for theoretical calculations or ideal condition.



Irreversibility

The mixing of two substances and combustion also lead to irreversibility. All spontaneous processes are irreversible.

Causes of Irreversibility

The irreversibility of a process may be due to the following reasons.

- 1) Lack of equilibrium during the process
 - i) Heat transfer due to lack of equilibrium
 - ii) Lack of pressure equilibrium with the interior of the system or between the system
 - iii) Free expansion
2. Irreversibility due to dissipative effects.

i) Friction

ii) Paddlewheel work transfer

iii) Transfer of electricity through a resistor

Carnot Cycle: Carnot cycle is a reversible cycle that is composed of four reversible processes, two isothermal and two adiabatic.

→ Process 1-2 (Reversible Isothermal Heat Addition)

$$Q_1 = W_{1-2} = mRT_1 \ln\left(\frac{V_2}{V_1}\right)$$

→ Process 2-3 (Reversible Adiabatic Expansion)

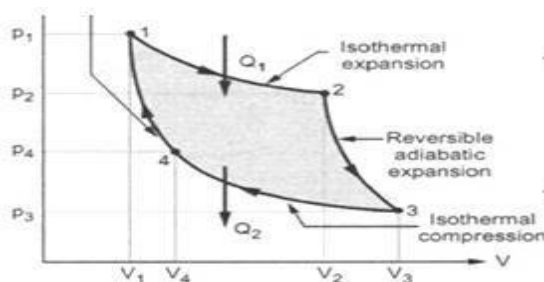
$$\frac{V_3}{V_4} = \frac{V_2}{V_1}; \quad \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \text{constant}$$

→ Process 3-4 (Reversible Isothermal Heat Rejection)

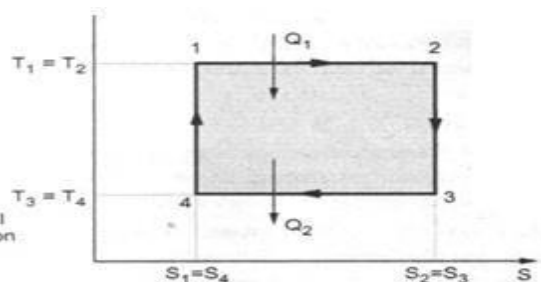
$$Q_2 = W_{3-4} = mRT_2 \ln\left(\frac{V_3}{V_4}\right)$$

→ Process 4-1 (Reversible Adiabatic Compression)

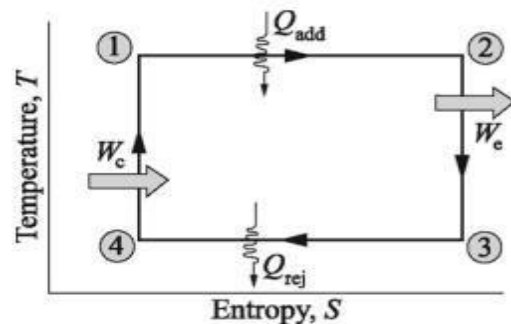
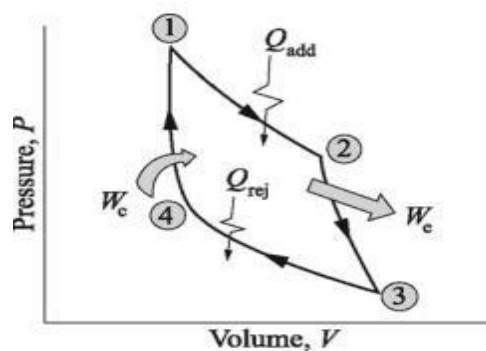
$$\frac{V_3}{V_4} = \frac{V_2}{V_1}; \quad \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \text{constant}$$



(a) p-V diagram



(b) T-S diagram



In Isothermal process

$$P_1 V_1 = P_2 V_2 \dots \dots \dots (1)$$

$$P_2 V_2^\gamma = P_3 V_3^\gamma \dots \dots \dots (2)$$

$$P_3 V_3^\gamma = P_4 V_4^\gamma$$

$$P_4 V_4^\gamma = P_1 V_1^\gamma$$

Multiplying (1)(2)(3) and (4):

$$V_2^{\gamma-1} V_4^{\gamma-1} = V_1^{\gamma-1} V_3^{\gamma-1}$$

$$V_2 V_4 = V_1 V_3$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$\log \frac{V_2}{V_1} = \log \frac{V_3}{V_4}$$

Dividing (4) and (2)

$$\frac{Q_1}{Q_2} = \frac{KT \log \frac{V_3}{V_4}}{KT \log \frac{V_1}{V_2}}$$

$$\frac{Q_1}{Q_2} = \frac{T_2}{T_1}$$

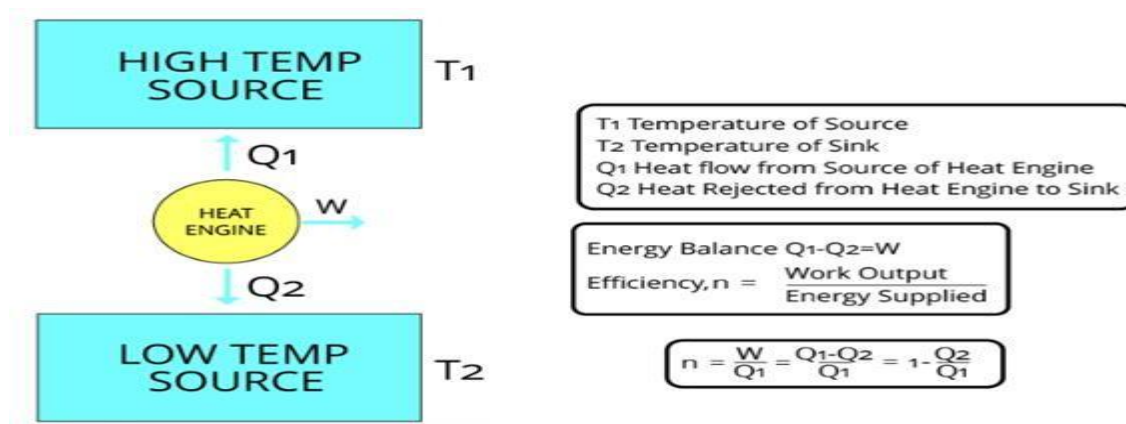
$$\eta_{car} = 1 - \frac{T_2}{T_1}$$

Heat engines that are working between two heat reservoirs are less efficient than the Carnot heat engine that is operating between the same reservoirs.

Carnot's Theorem Proof

Consider a heat engine that draws heat Q_1 from a heat reservoir at T_1 delivers work and dumps heat Q_2 into a heat at T_2

The heat engine operates in cycles (each of which includes two isothermal and two adiabatic processes), meaning it takes in heat Q_1 , performs work W , dumps heat Q_2 and then returns to its original unaltered state



Consider the universe's net change in entropy ΔS

At a steady temperature T_1 , the heat reservoir emits Q_1 , As a result, the change in its entropy is

$$\Delta S_1 = \frac{Q_1}{T_1}$$

At a steady temperature T_2 , the heat reservoir emits Q_2 , As a result, the change in its entropy is

$$\Delta S_2 = \frac{Q_2}{T_2}$$

As a result, the universe's net entropy change is

$$\Delta S = \Delta S_1 + \Delta S_2$$

$$\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1}$$

Using the second law of thermodynamics, $\Delta S \geq 0$, it follows that

$$\frac{Q_2}{T_2} - \frac{Q_1}{T_1} \geq 0$$

$$\frac{Q_2}{T_2} \geq \frac{Q_1}{T_1}$$

$$\frac{Q_2}{Q_1} \geq \frac{T_2}{T_1}$$

$$1 - \frac{Q_2}{Q_1} \leq 1 - \frac{T_2}{T_1}$$

Because the left side reflects the efficiency of the supplied heat engine η the right side indicates the efficiency of a Carnot engine

$$\eta \leq \eta_{\text{Carnot}} \rightarrow \eta_{\text{max}} = \eta_{\text{Carnot}}$$

Hence proved

Example 1: An inventor claims that this engine has the following specifications :

Temperature limits 750°C and 25°C

Power developed 75 kW

Fuel burned per hour 3.9 kg

Heating value of the fuel 74500 kJ/kg

State whether his claim is valid or not.

To find

$$\text{i) } \eta_{\text{carnot}} \qquad \text{ii) } \eta_{\text{thermal}} = \frac{\text{Heat supplied}}{\text{Work done}}$$

Solution.

Temperature of source, $T_1 = 750 + 273 = 1023\text{ K}$

Temperature of sink, $T_2 = 25 + 273 = 298\text{ K}$

$$\eta_{\text{carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{298}{1023} = \mathbf{0.7086 \text{ or } 70.86\%}$$

The actual thermal efficiency claimed,

$$\eta_{\text{thermal}} = \frac{\text{Heat supplied}}{\text{Work done}} = \frac{75 \times 1000 \times 60 \times 60}{39 \times 74500 \times 1000} = \mathbf{0.9292 \text{ or } 92.92\%}.$$

Since

$$\eta_{\text{thermal}} > \eta_{\text{carnot}},$$

therefore claim of the inventor is **not valid (or possible)**. (Ans.)

Example 2: What is the highest possible theoretical efficiency of a heat engine operating with a hot reservoir of furnace gases at 2100°C when the cooling water available is at 15°C ?

To find: theoretical efficiency $\eta_{\text{theoretical}}$

Solution.

Temperature of furnace gases, $T_1 = 2100 + 273 = 2373\text{ K}$ Temperature

of cooling water, $T_2 = 15 + 273 = 288\text{ K}$

$$\begin{aligned} \text{Now, } \eta_{\text{max}} (= \eta_{\text{carnot}}) &= 1 - \frac{T_2}{T_1} \\ &= 1 - \frac{288}{2373} = \mathbf{0.878 \text{ or } 87.8\%}. (\text{Ans.}) \end{aligned}$$

Example3: A Carnot cycle operates between source and sink temperatures of 250°C and -15°C . If the system receives 90kJ from the source, find :

- (i) Efficiency of the system; $\eta_{\text{carnot}} = 1 - \frac{T_2}{T_1}$
- (ii) The network transfer; $[\eta_{\text{carnot}} = \frac{W}{Q_1}]$
- (iii) Heat rejected to sink. $W = Q_1 - Q_2$

Solution. Temperature of source, $T_1 = 250 + 273 = 523\text{K}$

Temperature of sink, $T_2 = -15 + 273 = 258\text{K}$

Heat received by the system, $Q_1 = 90\text{kJ}$

$$(i) \quad \eta_{\text{carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{288}{523} = 0.506 = 50.6\%. (\text{Ans.})$$

ii) The network transfer,

$$W = \eta_{\text{carnot}} \times Q_1 \quad \left[\eta_{\text{carnot}} = \frac{W}{Q_1} \right]$$

$$= 0.506 \times 90 = 45.54\text{kJ}. (\text{Ans.})$$

(iii) Heat rejected to the sink, Q_2

$$= Q_1 - W \quad [W = Q_1 - Q_2]$$

$$= 90 - 45.54 = 44.46\text{kJ}. (\text{Ans.})$$

Example4: A fish freezing plant requires 40 tons of refrigeration. The freezing temperature is -35°C while the ambient temperature is 30°C . If the performance of the plant is 20% of the theoretical reversed Carnot cycle working within the same temperature limits, calculate the power required.

Given: 1 ton of refrigeration = 210kJ/min .

Solution.

Cooling required = 40 tons = $40 \times 210 = 8400\text{kJ/min}$

Ambient temperature, $T_1 = 30 + 273 = 303\text{K}$

Freezing temperature, $T_2 = -35 + 273 = 238\text{K}$

Performance of plant = 20% of the theoretical reversed Carnot cycle

$$(\text{C.O.P.})_{\text{refrigerator}} = \frac{T_2}{T_1 - T_2} = \frac{238}{303 - 238} = 3.66$$

$$\therefore \text{Actual C.O.P.} = 0.20 \times 3.66 = 0.732$$

$$(\text{C.O.P.})_{\text{actual}} = \frac{\text{Cooling reqd.}}{\text{Work needed}} \quad 0.732 = \frac{8400}{W} \quad \text{or } W = \frac{8400}{0.732} \text{ kJ/min}$$

$$= 191.25 \text{ kJ/s} = 191.25 \text{ kW}$$

Hence, power required = 191.25 kW. (Ans.)

*****Example 5.** A reversible heat engine operates between two reservoirs at temperatures 700°C and 50°C . The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 50°C and -25°C . The heat transfer to the engine is 2500 kJ and the net work output of the combined engine refrigerator plant is 400 kJ . (i) Determine the **heat transfer** to the refrigerant and the **net heat transfer** to the reservoir at 50°C ; (ii) Reconsider (i) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 45 per cent of their maximum possible values.

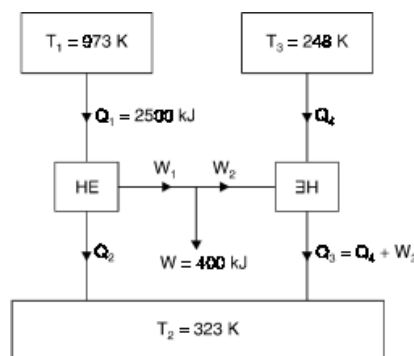
Given:

$$\text{Temperature, } T_1 = 700 + 273 = 973 \text{ K}$$

$$\text{Temperature, } T_2 = 50 + 273 = 323 \text{ K}$$

$$\text{Temperature, } T_3 = -25 + 273 = 248 \text{ K}$$

$$\text{The heat transfer to the heat engine, } Q_1 = 2500 \text{ kJ}$$



Solution.

The net work output of the combined engine refrigerator plant,

$$W = W_1 - W_2 = 400 \text{ kJ.}$$

(i) Maximum efficiency

$$\eta_{\max} = 1 - \frac{T_2}{T_1} = 1 - \frac{323}{973} = 0.668$$

Efficiency Carnot engine

$$\eta_{\text{carnot}} = \frac{W}{Q_1} = 0.668$$

$$\therefore W_1 = 0.668 \times 2500 = 1670 \text{ kJ}$$

Coefficient of Performance (C.O.P)

$$(\text{C.O.P.})_{\max} = \frac{T_3}{T_2 - T_3} = \frac{248}{323 - 248} = 3.306$$

$$\text{Also, C.O.P.} = \frac{Q_4}{W} = 3.306$$

Since, $W = 400 \text{ kJ} = W_1 - W_2$

$$W_2 = W_1 - W = 1670 - 400 = 1270 \text{ kJ}$$

$$\frac{Q_4}{W} = 3.306 \quad ; \quad Q_4 = 3.306 \times W$$

$$\therefore Q_4 = 3.306 \times 1270 = 4198.6 \text{ kJ}$$

$$Q_3 = Q_4 + W_2 = 4198.6 + 1270 = 5468.6 \text{ kJ} \quad Q_2 =$$

$$Q_1 - W_1 = 2500 - 1670 = 830 \text{ kJ.}$$

Heat rejection to the 50°C reservoir

$$= Q_2 + Q_3 = 830 + 5468.6 = 6298.6 \text{ kJ. (Ans.)}$$

(ii) Efficiency of actual heat engine cycle, $\eta = 0.45$

$\eta = 45\%$ percent of their maximum possible values. $\eta =$

$$0.45 \times \eta_{\max}$$

$$\eta = 0.45 \times 0.668 = 0.3$$

$$\eta_{\text{carnot}} = \frac{W}{Q_1}$$

$$\therefore W_1 = \eta \times Q_1 = 0.3 \times 2500 = 750 \text{ kJ } W_2 =$$

$$W_1 - W$$

$$\therefore W_2 = 750 - 400 = 350 \text{ kJ}$$

C.O.P. of the actual refrigerator cycle,

$$\text{C.O.P.} = \frac{Q_4}{W} = 0.45 \times 3.306 = 1.48$$

$$\therefore Q_4 = 350 \times 1.48 = 518 \text{ kJ. (Ans.) } Q_3 =$$

$$518 + 350 = 868 \text{ kJ}$$

$$Q_2 = 2500 - 750 = 1750 \text{ kJ}$$

Heat rejected to 50°C reservoir

$$= Q_2 + Q_3 = 1750 + 868 = 2618 \text{ kJ. (Ans.)}$$

Example 6. A heat pump is to be used to heat a house in winter and then reversed to cool the house in summer. The **interior temperature** is to be maintained at 20°C. Heat transfer through the walls and roof is estimated to be 0.525 kJ/s per degree temperature difference between the inside and outside. (a) If the outside temperature in winter is 5°C, what is the minimum power required to drive the heat pump? (b) If the power output is the same as in part (a), what is the maximum outer temperature for which the inside can be maintained at 20°C?

Given:

$$\text{Source temperature} = 20^\circ\text{C} = 293 \text{ K}$$

$$\text{outside temperature} = 5^\circ\text{C} = 278 \text{ K Heat}$$

$$\text{transfer} = 0.525 \text{ kJ/s}$$

To find

i) what is the minimum power required

- ii) maximum outertemperature for which the inside can be maintained at 20°C?

Solution.

Calculate minimum power required

$$\begin{aligned} \text{a) Estimated Heat rate } Q &= q(T_1 - T_2) \\ &= 0.525 \times (20 - 5) \text{ kJ/s} = 7.875 \text{ kJ/s} \end{aligned}$$

$$\text{COP} = \frac{293}{293 - 278} = 19.53$$

$$W_{\min} = \frac{Q}{\text{COP}_{\max}} = \frac{7.875}{19.53} = 0.403 \text{ kW}$$

$$\text{minimum power required} = 403 \text{ W}$$

(b) Calculate maximum outertemperature

$$\text{Given } W = 403 \text{ W}$$

$$\begin{aligned} \text{Heat rate (Q)} &= 0.525 (T - 293) \text{ kW} \\ &= 525 (T - 293) \text{ W} \end{aligned}$$

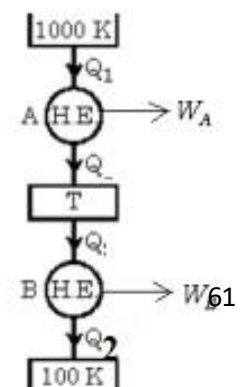
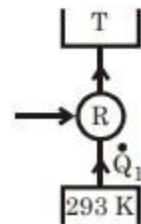
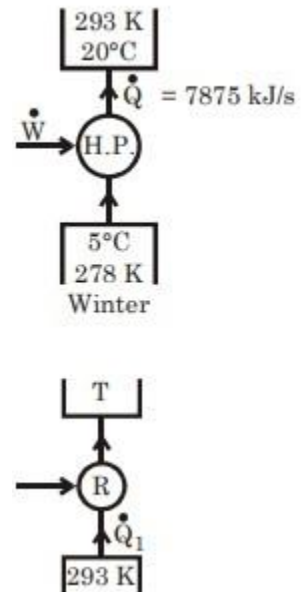
$$\therefore \text{COP} = \frac{525(T - 293)}{403} = \frac{293}{(T - 293)}$$

$$(T - 293) = \frac{403 \times 293}{525} = 280$$

$$T = 308 \text{ K} = 35^\circ\text{C}$$

$$\therefore \text{Maximum outside Temperature} = 35^\circ\text{C}$$

Example 7 . Two Carnot engines **A** and **B** are connected in **series** between two thermal reservoirs maintained at 1000 K and 100 K respectively. Engine **A** receives **1680 kJ of heat** from the high-temperature reservoir and rejects heat to the Carnot engine B. Engine B takes in heat rejected by engine A and rejects heat to the low-temperature reservoir. If engines A and B have equal thermal efficiencies, determine (a) The heat rejected by engine B (b) The temperature at which heat is rejected by engine, A (c) The work done



during the process by engines, A and B respectively. If engines A and B deliver equal work, determine (d) The amount of heat taken in by engine B (e) The efficiencies of engines A and B

Given:

$$T_{\text{High}} = 1000 \text{ K}$$

$$T_{\text{Low}} = 100 \text{ K}$$

$$Q_{\text{High}} = 1680 \text{ kJ}$$

To find:

Case: 1 A and B have equal thermal efficiencies,

Calculate

i) heat rejected by engine B

ii) work done

Case: 2 A and B have delivered equal work,

Amount of heat taken in by B η_A

and η_B

Solution:

Based on efficiencies $\eta_A = \eta_B$

$$\eta_A = \frac{W_A}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{1000}$$

$$\eta_B = \frac{W_B}{Q} = 1 - \frac{Q_2}{Q} = 1 - \frac{T_2}{T} = 1 - \frac{100}{T}$$

$$1 - \frac{100}{1000} = 1 - \frac{100}{T}$$

$$T = \sqrt{1000 \times 100}$$

$$T = 316.3 \text{ K}$$

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}; Q_2 = Q_1 \times \frac{T_2}{T_1} = 1680 \times \frac{100}{1000}$$

$$Q_2 = 168 \text{ kJ}$$

$$\frac{Q_2}{Q_1} = \frac{Q_2}{Q} \Rightarrow Q = \frac{Q_2}{\frac{Q_2}{Q_1}} = \frac{168 \times 1000}{316.3} = 531.26 \text{ kJ}$$

$$Q_2 = 168 \text{ kJ}$$

$$W_A = Q_1 - Q_2 = (1880 - 531.26) \text{ kJ}$$

$$= 1148.74 \text{ kJ}$$

$$W_B = (531.26 - 168) \text{ kJ} = 363.26 \text{ kJ}$$

If the equal work

$$T = \frac{100 + 1000}{2} = 550 \text{ K}$$

$$\frac{Q}{T} = \frac{Q_1}{T_1}$$

$$\therefore Q = \frac{Q_1}{T_1} \times T = \frac{1680}{1000} \times 550 = 924 \text{ kJ}$$

Efficiencies of engines A and B

$$\eta_A = 1 - \frac{T_2}{T_1} = 1 - \frac{550}{1000} = 0.45$$

$$\eta_B = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{550} = 0.8182$$

*****Example 8.** A heat pump working on the **Carnot cycle** takes in **heat from a reservoir at 5°C** and delivers heat to a **reservoir at 60°C**. The heat pump is driven by a reversible heat engine which takes in heat from a **reservoir at 840°C** and rejects heat to a **reservoir at 60°C**. The reversible heat engine also drives a machine that **absorbs 30 kW**. If the heat pump extracts **17 kJ/s** from the 5°C reservoir, determine (a) The rate of heat supply from the 840°C source (b) The rate of heat rejection to the 60°C sink.

Given:

$$\text{reservoir} = 840 + 273 = 1113 \text{ K} [T_1]$$

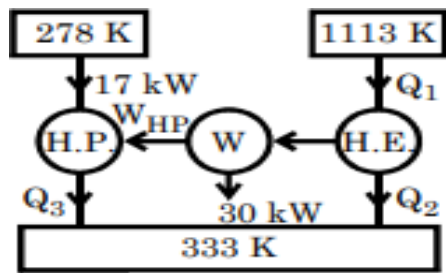
$$\text{Cycle takes in heat at } 5^\circ\text{C} = 273 + 5 = 278 \text{ K} [T_4]$$

$$\text{Delivers reservoir at } 60^\circ\text{C} = 273 + 60 = 333 \text{ K} [T_2 = T_3]$$

$$\text{Heat pump extracts} = 17 \text{ kJ/s} = Q_4$$

$$\text{work down} = 30 \text{ kW}$$

To find: Solution:



Solution: **The rate of heat supply from the 840°C**

$$\text{COP of H.P.} = \frac{T_2}{T_2 - T_1} = \frac{333}{333 - 278} = 6.05454$$

$$\text{COP of H.P.} = \frac{Q_3}{Q_3 - 17} = 6.05454$$

$$Q_3 = 20.36 \text{ kW}$$

Work input to the heat pump $W_2 = Q_3 - Q_4 = 20.36 - 17 = 3.36 \text{ kW}$

$\therefore$ Work output of the Heat engine $W_{\text{H.E.}} = 30 + 3.36 = 33.36 \text{ kW}$

Maximum efficiency of heat engine $[\eta] = 1 - \frac{333}{1113} = 0.7$

$$\eta_{\text{Maxi}} = \frac{W_1}{Q_1} = \frac{W_1}{1} = \frac{333}{0.7008} = 47.6 \text{ kW}$$

$$W_1 = Q_1 - Q_2$$

Heat rejection to sink.

$$Q_2 = Q_1 - W_1 = 47.6 - 33.36 = 14.24 \text{ kW}$$

Net heat transferred to reservoir at 60°

$$Q_2 + Q_3 = 14.24 + 20.36 = 34.6 \text{ kW}$$

*****Example 9** .A heat engine operating between two reservoirs at 1000 K and 300 K is used to drive a heat pump which extracts heat from the reservoir at 300 K at a rate twice that at which the engine rejects heat to

it. If the efficiency of the engine is 40 % of the maximum possible and the cop of the heat pump is 50 % of the maximum possible, what is the heat temperature of the reservoir to which the heat pump rejects heat? What is the rate of heat rejection from the heat pump if the rate of heat supply to the engine is 50 kW ?

Sol.: Given data:

$$T_1 = 1000 \text{ K}, T_2 = 300 \text{ K}$$

$$\text{actual } \eta_{\text{engine}} = 40\% \eta_{\text{car}}$$

$$\text{COP}_{\text{max}} = 50\% \text{COP}$$

$$Q_1 = \text{supply to the engine is } 50 \text{ kW}$$

To find Carnot efficiency

$$\eta_{\text{carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{1000}$$

$$\eta_{\text{carnot}} = 0.7$$

$$\text{actual } \eta_{\text{engine}} = 40\% \eta_{\text{car}}$$

$$= 0.4 \times 0.7 = 28\%$$

Rate of heat rejection from the heat engine

$$\text{actual } \eta_{\text{engine}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$0.28 = \frac{50 - Q_2}{50}$$

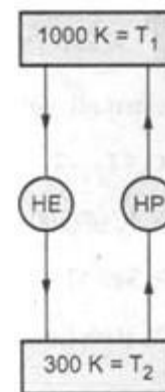
$$Q_2 = 36 \text{ kW}$$

Reservoir heat reject = twice the engine rejects

$$Q_3 = 2Q_2$$

$$Q_3 = 2 \times 36 = 72 \text{ kW}$$

Net Work down



$$(W)_{H.E} = \text{Heatsupplied}(Q_1) - \text{Heatrejected}(Q_2)$$

$$= 50 - 36 = 14 \text{ kW}$$

Calculate heat temperature of the reservoir heat pump projects

$$\text{cop}_{HP} = \frac{Q_3}{Q_1 - Q_2} = \frac{86}{86 - 72} = 6.14$$

$$\text{Actual cop}_{HP} = 0.5 \text{ cop}_{\text{carnot}}$$

$$6.14 = 0.5 \times \frac{T_3}{T_3 - T_2}$$

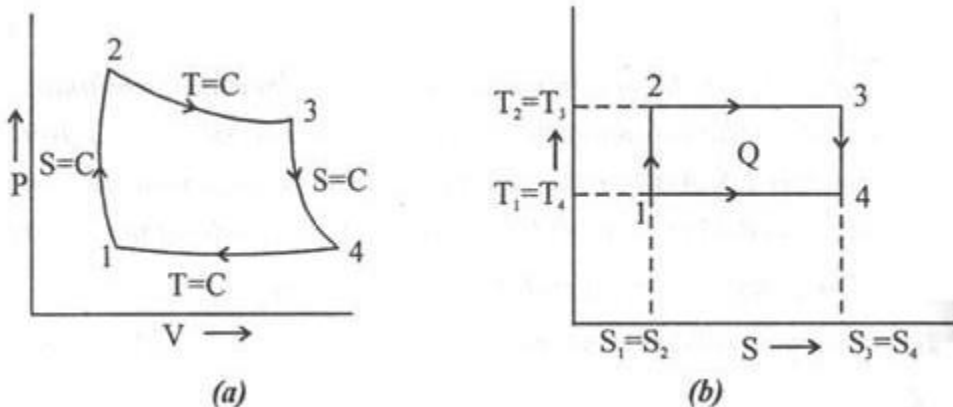
$$6.14 = 0.5 \times \frac{T_3}{T_3 - 300}$$

$$T_4 = 326.67 \text{ K}$$

*****Example 10. In a Carnot cycle the maximum pressure & temperature are limited to 18 bar & 410°C. The volume ratio of isentropic compression is 6 & isothermal expansion is 1.5. Assume the volume of air at the beginning of isothermal expansion as 0.18 m³. Show the cycle on p-V & T-s diagrams & determine**

The pressure & temperature at main points

Thermal efficiency of the cycle



Given data:

$$\text{Maximum pressure, } P_2 = 18 \text{ bar} = 18 \times 100$$

$$=1800\text{kN/m}^2 \text{ or } \text{kPa}$$

Maximum Temperature, $T_2 = 410^\circ\text{C}$

$$=410 + 273$$

$$=683\text{K} = T_3$$

Volume at the beginning of isothermal expansion $V_2 = 0.18 \text{ m}^3$

$$\text{Volume ratio for isothermal compression} = \frac{V_1}{V_2} = 6$$

$$\text{Volume ratio for isothermal expansion} = \frac{V_5}{V_2} = 1.5$$

To find:

Pressure and temperature at main point - $P_1, P_3 \& P_4$ and T_1, T_3 Thermal

efficiency ' n_{her} '

Solution: From p - V - T relation for an isentropic process,

Calculate the temperature at main point T_1, T_2, T_3, T_4 .

PROCESS 1-2 isothermal compression

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_1 = \frac{T_2}{\left(\frac{V_1}{V_2}\right)^{\gamma-1}} = \frac{638}{6^{1.4-1}} = 333.55$$

From T - S diagram $T_1 = T_4; T_2 = T_3$;

Calculate Pressure at main point

$$\left(\frac{V_1}{V_2}\right)^\gamma = \frac{P_2}{P_1}; P_1 = \frac{P_2}{\left(\frac{V_1}{V_2}\right)^\gamma} = \frac{1800}{6^{1.4}} = 146 \text{ kPa}$$

Process 2-3 for an isothermal Process

$$p_3 V_3 = p_2 V_2$$

$$P_3 = P_2 \left(\frac{V_2}{V_3} \right)^{\frac{1}{1.5}} = 1800 \times \frac{1}{1.5} = 1200 \text{ kPa}$$

Process 3-4 Reverse Adiabatic Process

$$p_3 V_3^\gamma = p_4 V_4^\gamma$$

$$p_4 = p_3 \left(\frac{V_3}{V_4} \right)^\gamma = 1200 \times \frac{1^{1.4}}{6}$$

$$p_4 = 97.7 \text{ kPa}$$

Calculate the Thermal efficiency 'n_{her}' $1 - \frac{T_1}{T_4}$

$$= 1 - \frac{333.5}{683} = 51.79\%$$

*****Example 11:** *Source A' can supply energy at a rate of 11,000 kJ/min at 320°C. Another source 'B' can supply energy at a rate of 11,000 kJ/min at 68°C. Which source 'A' or 'B' would you choose to supply energy to an ideal reversible engine that is to produce large amount of power, if the temperature of the surroundings is 40°C*

Given data:

Temperature of the surroundings (or) sink, T_t = 40°C

Source A:

$$= 313 \text{ K}$$

Heats supplied, Q_{S1} = 11,000 kJ/min = 183.33 kJ/s Source

temperature, T_{H1} = 320° C = 593 K

Source B:

Heats supplied, Q_{S2} = 11,000 kJ/min = 183.33 kJ/s Source

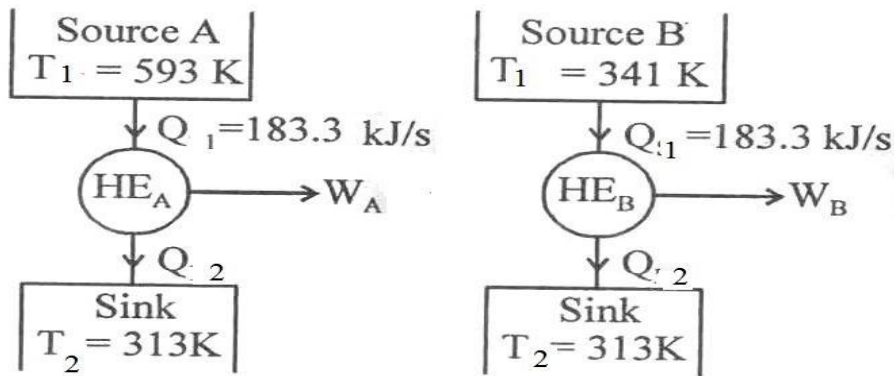
temperature, T_{H2} = 68°C = 341 K

To find:

Source which produces highest power 'W'

Case A :

For a reversible engine



$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1} = \frac{Q_2}{T_1} = \frac{T_2}{T_1}, \quad \frac{Q_2}{183.33} = \frac{313}{593}$$

$$Q_2 = 96.77 \text{ kJ/s}$$

By energy balance $W_A = Q_1 - Q_2 = 183.33 - 96.77 = 86.56 \text{ kJ/s}$

Case B:

For a reversible engine

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1} = \frac{Q_2}{T_1} = \frac{T_2}{T_1}, \quad \frac{Q_2}{183.33} = \frac{313}{341}$$

$$Q_2 = 168.28 \text{ kJ/s}$$

By energy balance $W_B = Q_1 - Q_2 = 183.33 - 168.28 = 15.05 \text{ kJ/s}$

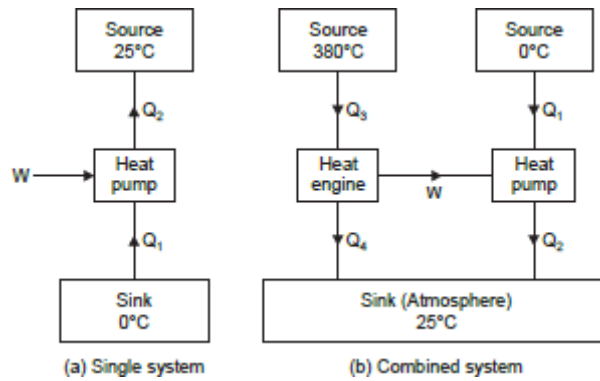
Here, $W_A > W_B$ So, source A is preferred

Example 12: A reversible heat pump is used to maintain a temperature of 0°C in a refrigerator when it rejects the heat to the surroundings at 25°C. If the heat removal rate from the refrigerator is 1440 kJ/min, determine the C.O.P. of the machine and work input required. (ii) If the required input to run the pump is developed by a reversible engine which receives heat at 380°C and rejects heat to atmosphere, then determine the overall C.O.P. of the system.

Solution.

(i) Temperature, $T_1 = 25 + 273 = 298 \text{ K}$

Temperature, $T_2 = 0 + 273 = 273 \text{ K}$



Heat removal rate from the refrigerator,

$$Q_1 = 1440 \text{ kJ/min} = 24 \text{ kJ/s}$$

Now, co-efficient of performance, for reversible heat pump,

$$\text{C.O.P.} = \frac{T_1}{T_1 - T_2} = \frac{298}{(298 - 273)} = \mathbf{11.92. (Ans.)}$$

$$\therefore (\text{C.O.P.})_{\text{ref.}} = \frac{T_2}{T_1 - T_2} = \frac{273}{(298 - 273)} = 10.92$$

$$\text{Now, } 10.92 = \frac{Q_1}{W} = \frac{24}{W}$$

$$\therefore W = 2.2 \text{ kW}$$

i.e., Work input required = 2.2 kW. (Ans.)

$$Q_2 = Q_1 + W = 24 + 2.2 = 26.2 \text{ kJ/s}$$

(ii) Refer Fig. (b).

The overall C.O.P. is given by,

$$\text{C.O.P.} = \frac{\text{Heat removed from the refrigerator}}{\text{Heat supplied from the source}}$$

$$\frac{Q_1}{Q_3}$$

For the reversible engine, we can write

$$\frac{Q_3}{T_3} = \frac{Q_4}{T_4}$$

$$\frac{Q_3 + W}{T_3} = \frac{Q_4}{T_4}$$

$$\frac{Q_3 + 2.2}{653} = \frac{Q_4}{298}$$

$$298(Q_3 + 2.2) = 653Q_4$$

$$Q_4(653 - 298) = 298 \times 2.2$$

$$Q_4 = \frac{298 \times 2.2}{(653 - 298)} = 1.847 \text{ kJ/s}$$

$$\therefore Q_3 = Q_4 + W = 1.847 + 2.2 = 4.047 \text{ kJ/s}$$

Substituting this value in eqn. (i), we get

$$\text{C.O.P.} = \frac{24}{4.047} = 5.93. (\text{Ans.})$$

If the purpose of the system is to supply the heat to the sink at 25°C , then

$$\text{Overall C.O.P.} = \frac{Q_2 + Q_4}{Q_3} = \frac{26.2 + 1.87}{4.047} = 6.93. (\text{Ans.})$$

***Inequality in Clausius theorem

As the efficiency of a reversible Carnot cycle is,

$$\eta = 1 - \frac{d'Q_2}{d'Q_1} = 1 - \frac{T_2}{T_1}$$

efficiency of reversible engine (R) is greater than the irreversible engine when they operate at same temperatures.

$$\left(1 - \frac{d'Q_2}{d'Q_1}\right)_R \geq \left(1 - \frac{d'Q_2}{d'Q_1}\right)_I$$

$$1 - \frac{T_2}{T_1} \geq \left(1 - \frac{d'Q_2}{d'Q_1}\right)_I$$

$$\frac{T_2}{T_1} \leq \frac{d'Q_2}{d'Q_1}$$

$$\frac{d'Q_1}{T_1} \leq \frac{d'Q_2}{T_2}$$

$$\frac{d'Q_1}{T_1} - \frac{d'Q_2}{T_2} \leq 0$$

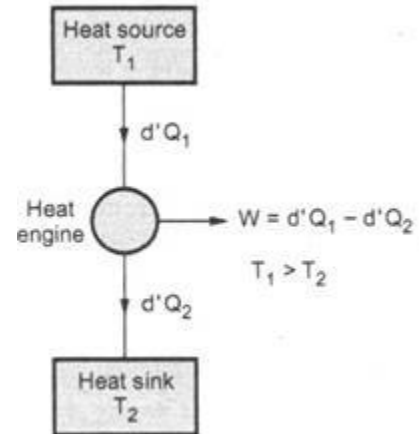
$$\frac{d'Q_1}{T_1} - \left(-\frac{d'Q_2}{T_2}\right) \leq 0$$

$$\frac{d'Q_1}{T_1} + \frac{d'Q_2}{T_2} \leq 0$$

$\oint \frac{d'Q}{T} \leq 0$ this equation is called Inequality in Clausius theorem

For reversible cycle $\oint \frac{d'Q}{T} = 0$

For irreversible cycle $\oint \frac{d'Q}{T} < 0$



$$\oint \frac{d'Q}{T} > 0 \text{ (not possible)}$$

***** Entropy Change for and Reversible Adiabatic and Reversible isothermal process**

Reversible adiabatic process

• For a reversible adiabatic process heat transfer between the system and surrounding is zero.

$$ds = \left(\frac{d'Q}{T} \right)_{\text{Rev adiabatic}} = 0$$

$S = \text{constant}$

Reversible isothermal process

• For a reversible isothermal process, the temperature T of the system is constant. Hence, the entropy change for an isothermal process is given by,

$$(ds) = \left(\oint \frac{d'Q}{T} \right)_R = \frac{1}{T} \int_R d'Q$$

$$ds = \frac{Q}{T}$$

• It means, the entropy change in an isothermal process is the ratio of heat transfer to the temperature of the system during the process.

***** Principle of increase in entropy**

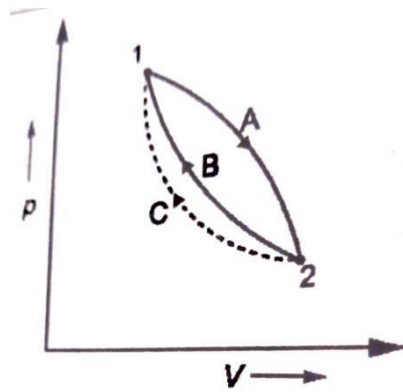
The principle of increase of entropy states that the entropy of the Universe also increases. It never decreases but remains constant only in a reversible process.

The entropy change of a system is given by the equation:

$$\Delta S = \frac{dQ}{T}$$

Where, dQ is the change of heat of the system and T is the temperature of the system.

Change in entropy is examined for an irreversible process



For the reversible cycle 1-A-2-B-1,

$$\int_{1A}^{2A} \frac{dQ}{T} + \int_{2B}^{1B} \frac{dQ}{T} = 0 \dots\dots\dots 1$$

For the irreversible cycle 1-A-2-C-1,

$$\int_{1A}^{2A} \frac{dQ}{T} + \int_{2C}^{1C} \frac{dQ}{T} = 0 \dots\dots\dots 2$$

Subtracting the equation 1 from 2

$$\int_{2C}^{1C} \frac{dQ}{T} - \int_{2B}^{1B} \frac{dQ}{T} \leq 0 \dots\dots\dots 3$$

On reversing the limit and rearranging,

$$\int_{2B}^{1B} \frac{dQ}{T} \geq \int_{2C}^{1C} \frac{dQ}{T}$$

Since, process 2-B is reversible, $dS = \frac{dQ}{T}$

Substituting this value in equation 3

$$\int_1^2 ds \geq \int_{1C}^{2C} \frac{dQ}{T} \text{ or } \Delta S \geq \frac{Q}{T}$$

For an isolated system:

$\Delta S > 0$, for irreversible processes

$\Delta S = 0$, for reversible processes

$\Delta S < 0$, this is an impossible case

******Example 10.** 300 kJ/s of heat is supplied at a constant fixed temperature of 290°C to a heat engine. The heat rejection takes place at 8.5°C. The following results were obtained:

- (i) 215 kJ/s are rejected.

(ii) 150kJ/s are rejected

(iii) 75kJ/s are rejected.

Classify which of the results report a reversible cycle or irreversible cycle or impossible results.

Solution.

Heat supplied at $290^{\circ}\text{C} = 300\text{kJ/s}$ Heat

rejected at 8.5°C :

(i) 215kJ/s,

(ii) 150kJ/s,

(iii) 75 kJ/s.

Applying Clausius inequality to the cycle or process, we

have :

$$(i) \text{ cycle } \sum \frac{\delta Q}{T} = \frac{300}{290+273} - \frac{215}{85+273}$$

$$= 0.5328 - 0.7637 = -0.2309 < 0.$$

$\therefore$ Cycle is irreversible. (Ans.)

$$(ii) \sum \frac{\delta Q}{T} = \frac{300}{290+273} - \frac{150}{85+273}$$

$$= 0.5328 - 0.5328 = 0$$

$\therefore$ Cycle is reversible. (Ans.)

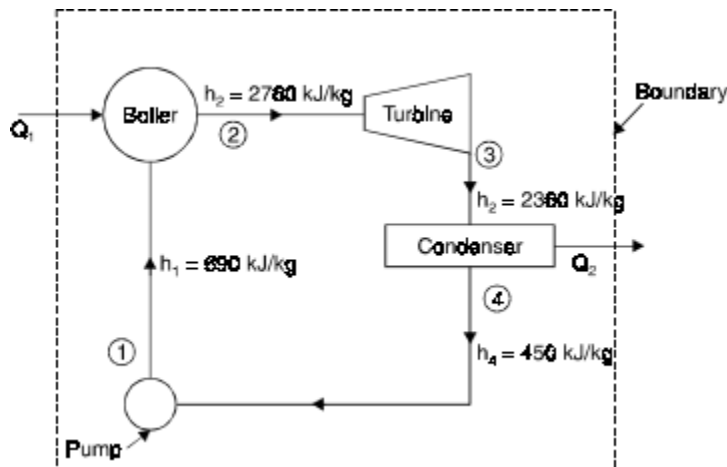
$$(iii) \sum \frac{\delta Q}{T} = \frac{300}{290+273} - \frac{75}{85+273}$$

$$= 0.5328 - 0.2664 = 0.2664 > 0.$$

This cycle is impossible by second law of thermodynamics, i.e., Clausius inequality. (Ans.)

Example 11. In a power plant cycle, the temperature range is 164°C to 51°C , the upper temperature being maintained in the boiler where heat is received and the lower temperature being maintained in the

condenser where heat is rejected. All other processes in the steady flow cycle are adiabatic. The specific enthalpies at various points are given in Fig. Verify the Clausius Inequality



Solution. Temperature maintained in boiler, $T_1 = 164 + 273 = 437 \text{ K}$

Temperature maintained in condenser, $T_2 = 51 + 273 = 324 \text{ K}$

Heat transferred in the boiler per kg of fluid, $Q_1 =$

$$h_2 - h_1 = 2760 - 690 = 2070 \text{ kJ/kg}$$

Heat transferred out at the condenser per kg of fluid,

$$Q_2 = h_4 - h_3 = 450 - 2360 = -1910 \text{ kJ/kg}$$

Since there is no transfer of heat at any other point, we have per kg

$$\sum_{cycle} \frac{\delta Q}{T} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{2070}{437} + \left(-\frac{1910}{324} \right)$$

$$= 4.737 - 5.895 = -1.158 \text{ kJ/kgK} < 0.$$

The Clausius Inequality is proved. The steady flow cycle is obviously irreversible.

If the cycle is reversible between the same temperature limits and the heat supplied at higher temperature is same, the heat rejected can be calculated as follows :

$$\eta_{\text{reversible}} = 1 - \frac{T_2}{T_1} = 1 - \frac{324}{437} = 0.2586 \text{ or } 25.86\%$$

∴ Heat rejected per kg is given by

$$Q_2 = (1 - 0.2586) \times Q_1 = (1 - 0.2586) \times 2070 = 1534.7 \text{ kJ/kg } \delta Q_{T \text{ cycle}}$$

Σ

$$\Sigma_{\text{cycle}} \frac{\delta Q}{T} = \frac{2070}{437} - \frac{1534.7}{324} = 4.73 - 4.73 = 0$$

$$\Sigma_{\text{cycle}} \frac{\delta Q}{T} = \frac{Q_{\text{added}}}{T_{\text{source}}} + \frac{Q_{\text{rejected}}}{T_{\text{sink}}}$$

Thus Clausius Equality sign for a reversible engine is verified

Available energy (A.E.),

Sources of energy can be divided into two groups,

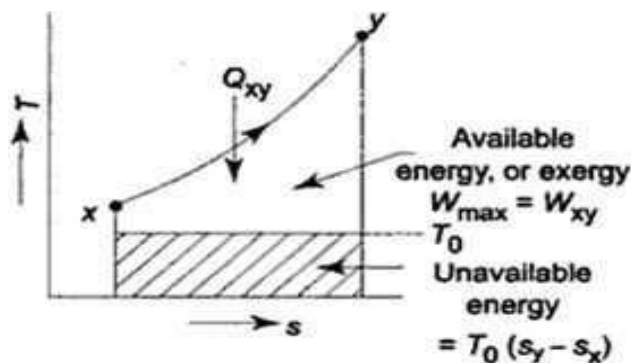
- high grade energy
- low grade energy

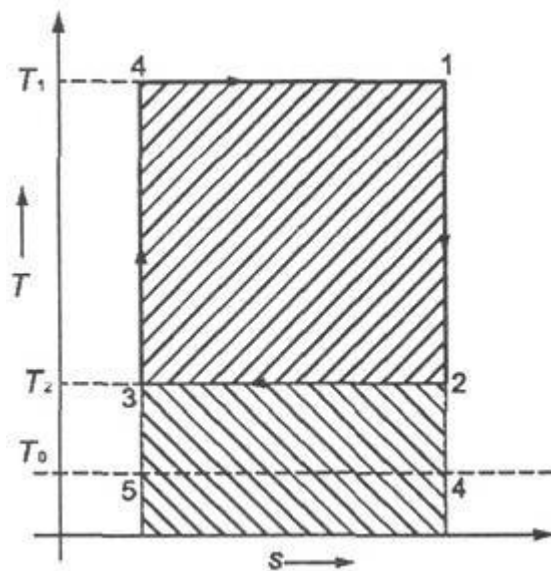
Available energy

The maximum work output obtainable from a certain heat input in a cyclic heat engine is called the *available energy* (A.E.),

Unavailable energy

The minimum energy that has to be rejected to the sink by the second law is called the *unavailable energy*





According to the second law of thermodynamics, the area of 1-2-3-4 is called *useful energy*. But, the area of 2-3-5-4 refers to the energy losses to the surroundings due to friction (uncontrollable loss).

Available energy:

$$A.E = Q - T_0 \Delta S \quad \left[\left(1 - \frac{T_0}{T_1} \right) Q; \Delta S = \frac{Q}{T_1} \right]$$

Q = Heat transfer T_1 = Source temperature

T_2 or T_0 = surrounding temperature Entropy principle ΔS

Unavailable energy:

$$U.A.E = Q - A.E \quad [Q - [(Q - T_0 \Delta S)]] \\ = T_0 \Delta S$$

Type of process	Available energy:	Unavailable energy:
Constant volume process	$A.E = Q - T_0 \Delta S$ where $\Delta S = m C_v \ln \frac{T_2}{T_1}$ $Q = m C_v (T_2 - T_1)$	$Q - A.E = T_0 \Delta S$ $U.A.E = T_0 \Delta S$
Constant pressure or isobaric process	$A.E = Q - T_0 \Delta S$ $Q = m C_p (T_2 - T_1)$	$U.A.E = T_0 \Delta S$

	$\Delta S = m C_p \ln \frac{T_2}{T_1}$	
Constant temperature or isothermal process	$A.E = Q - T_0 \Delta S$ $Q = p_1 V_1 \ln \left(\frac{p_2}{p_1} \right)$ $\Delta S = \frac{Q}{T_1} \text{ or } \frac{Q}{T_2}$	$U.A.E = T_0 \Delta S$
Polytropic Process	$Q = (\gamma - n) \frac{p_1 V_1 - p_2 V_2}{n - 1}$ $= (\gamma - n) \frac{m R (T_1 - T_2)}{n - 1}$ $\Delta S = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$ $\Delta S = C_v \ln \frac{T_2}{T_1} - R \ln \frac{V_2}{V_1}$ $A.E \text{ or } W_{max} = Q - T_0 \Delta S$	Irreversibility $I = W_{max} - W$ $= T_0 \Delta S$

Open System In Terms Of Availability And Second Law Efficiency

	Availability change	Irreversibility
Turbine:	$A.E = W_{max} = m(h_1 - h_2 - T_0(S_1 - S_2))$	$I = W_{max} - W_{act}$ $= T_0 \Delta S$
	Second law efficiency $= \eta_{(II)}$ $\eta_{(II)} = \frac{W_{max}}{A_{act}}$	
Compressor / pump	Maximum work input, [A.E] $W_{max} = m(h_2 - h_1 - T_0(S_2 - S_1))$	Irreversibility $I = W_{max} - W$ $= T_0 \Delta S$
	Second law efficiency $\eta_{(II)} = \frac{W_{max}}{A_{act}}$	
Heat Exchanger	Availability of cold fluid, $B_c = m_2 C_2 (T_4 - T_3) - T_o (S_4 - S_3)$ Availability of Hot fluid,	Irreversibility, $I = B_h - B_c$

	$Bh = m_1 C_2 (T_1 - T_2) - T_0 (S_1 - S_2)$ $\text{Second law efficiency} = \eta_{(II)} =$	
Throttling process	$W_{max} = (h_1 - h_2 - T_0 (S_1 - S_2))$ $W_{max} = (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$ $= \psi_1 - \psi_2$	

EFFECTIVENESS

Effectiveness is defined as the ratio of actual useful work to the maximum useful work.

$$\text{Effectiveness, } \epsilon = \frac{\text{Increase of availability of surroundings}}{\text{Loss of availability of the system}}$$

First law efficiency

First law of efficiency is defined as the ratio between network output and heat supplied to the system

$$\eta_{\text{Frist(I)}} = \frac{\text{network output}}{\text{heat supplied to the system}}$$

Second law efficiency is defined as the ratio between the change in the available energy the system and change in the available energy of the source

$$\eta_{(II)} = \frac{\text{Change in the available energy of the system}}{\text{Change in the available energy of the source}} = \frac{A_{out}}{A_{in}}$$

Open System In Terms Of Availability And Second Law Efficiency

Turbine:

Reversible work (or) maximum work or availability change

$$\text{availability change } A.E = W_{max} = m(h_1 - h_2 - T_0(S_1 - S_2))$$

$$\text{Irreversibility } I = W_{max} - W_{act} \\ = T_0 \Delta S$$

$$\text{Second law efficiency} = \eta_{(II)} = \frac{W_{max}}{A_{act}}$$

Compressor/pump

Maximum work input, [A.E]

$$W_{max} = m(h_2 - h_1 - T_0(S_2 - S_1))$$

Irreversibility

$$I = W_{max} - W \\ = T_0 \Delta S$$

Second law efficiency

$$= \eta_{(II)} = \frac{W_{max}}{A_{act}}$$

Heat Exchanger

Availability of cold fluid,

$$B_c = m_2 C_2 (T_4 - T_3) - T_0 (S_4 - S_3)$$

Availability, of Hot fluid,

$$B_h = m_1 C_1 (T_1 - T_2) - T_0 (S_1 - S_2)$$

Irreversibility,

$$I = B_h - B_c$$

Second law efficiency = $\eta_{(II)} = -$

Throttling process

$$W_{max} = (h_1 - h_2 - T_0(S_1 - S_2))$$

$$W_{max} = (h_1 - T_0 S_1) - (h_2 - T_0 S_2) = \psi_1 - \psi_2$$

EFFECTIVENESS

Effectiveness is defined as the ratio of actual useful work to the maximum useful work.

$$\text{Effectiveness, } \epsilon = \frac{\text{Increase of availability of surroundings}}{\text{Loss of availability of the system}}$$

Example 1: 15 kg of water is heated in an insulated tank by a churning process from 300 K to 340 K. If the surrounding temperature is 300 K, find the loss in availability for the process.

Solution.

Mass of water, $m = 15 \text{ kg}$

Temperature, $T_1 = 340 \text{ K}$

Surrounding temperature, $T_0 = 300 \text{ K}$

Specific heat of water, $c_p = 4.187 \text{ kJ/kgK}$

Loss in availability:

Work added during churning

= Increase in enthalpy of the water

$$= mc_v(T_2 - T_0)$$

$$= 15 \times 4.187 \times (340 - 300) = 2512.2 \text{ kJ}$$

energy in the water = 2512.2 kJ

availability out of this energy is given by

$$m(h_1 - h_2 - T_0(S_1 - S_2))$$

$$\text{where } \Delta s = c_p \log_e \frac{T_1}{T_0}$$

$$\therefore \Delta s = 4.187 \log_e \frac{340}{300}$$

$$= 0.524 \text{ kJ/kgK}$$

$\therefore$ Available energy

$$= m[c_v(T_1 - T_0) - T_0 \Delta s]$$

$$= 15[4.187(340 - 300) - 300 \times 0.524] = 158.7 \text{ kJ}$$

∴ Loss in availability

$$= 2508 - 158.7 = 2349.3 \text{ kJ. (Ans.)}$$

Example 2: One kg of air is compressed polytropically from 1 bar pressure and temperature of 300 K to a pressure of 6.8 bar and temperature of 370 K. Determine the irreversibility if the sink

temperature is 293 K. Assume $R = 0.287 \text{ kJ/kg K}$, $c_p = 1.004 \text{ kJ/kg K}$ and $c_v = 0.716 \text{ kJ/kg K}$.

Given:

$$p_1 = 1 \text{ bar: temperature } T_1 = 300 \text{ K}$$

$$p_2 = 6.8 \text{ bar: temperature } T_2 = 370 \text{ K}$$

$$T_0 = 293 \text{ K } R = 0.287 \text{ kJ/kg K}, c_p = 1.004 \text{ kJ/kg K } c_v = 0.716 \text{ kJ/kg K.}$$

$$\text{To find: irreversibility } I = W_{\max} - W_{\text{act}} \\ = W_{\max} = (h_1 - h_2 - T_0(S_1 - S_2))$$

Solution:

$$\text{Irreversibility } I = W_{\max} - W_{\text{act}}$$

$$-W_{\max} = \text{Change in internal energy} - T_0 \times \text{Change in entropy}$$

$$-W_{\max} = (h_1 - h_2 - T_0(S_1 - S_2))$$

$$-W_{\max} = c_v(T_2 - T_1) - T_0[c_p \ln(T_2/T_1) - R \ln(p_2/p_1)]$$

$$= 0.716(370 - 300) - 293 \times [1.005 \ln \frac{370}{300} - 0.287 \ln \frac{6.8}{1}]$$

$$W_{\max} = -149.53 \text{ kJ/kg} = W_{\text{rev}}$$

(negative sign indicates that work is done on air)

$$W_{\text{act}} = \frac{mR(T_1 - T_2)}{n - 1}$$

The index of compression 'n' is given by

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$

$$\frac{n-1}{n} = \frac{\ln \frac{370}{300}}{\ln \frac{6.8}{1}}$$

$$n = 1.123$$

$$W_{act} = \frac{mR(T_1 - T_2)}{n-1} = \frac{1 \times 0.287(370 - 300)}{1.123 - 1}$$

$$= -163.33 \text{ kJ/kg}$$

$$I = W_{rev} - W_{act} = -149.53 - (-163.33)$$

$$\text{Irreversibility} = 13.8 \text{ kJ/kg. (Ans.)}$$

Example 3: 8 kg of air at 650 K and 5.5 bar pressure is enclosed in a closed system. If the atmospheric temperature and pressure are 300 K and 1 bar respectively, determine :

(i) The availability if the system goes through the ideal work producing process.

(ii) The availability and effectiveness if the air is cooled at constant pressure to atmospheric temperature without bringing it to complete dead state. Take $c_v = 0.718 \text{ kJ/kgK}$; $c_p = 1.005 \text{ kJ/kg K}$.

Solution. Mass of air, $m = 8 \text{ kg}$

Temperature, $T_1 = 650 \text{ K}$

Pressure, $p_1 = 5.5 \text{ bar}$

Atmospheric pressure, $p_0 = 1 \text{ bar}$

Atmospheric temperature, $T_0 = 300 \text{ K}$

For air: $c_v = 0.718 \text{ kJ/kgK}$; $c_p = 1.005 \text{ kJ/kgK}$.

Change in availability A.E. $= W_{max} = m(h_1 - h_2 - T_0(S_1 - S_2))$ ($h_1 -$

h_2) $= c_v(T_1 - T_0)$

$$(S_1 - S_2) = \Delta S = C_v \ln \frac{T_2}{T_1} - R \ln \frac{V_1}{V_0}$$

Using the ideal gas equation, $\frac{p_1 v_1}{T_1} = \frac{p_0 v_0}{T_0} =$

$$\frac{v_0}{v_1} = \frac{p_1}{p_0} = \frac{T_0}{T_1} = \frac{5.5}{1} \times \frac{300}{650} = 2.54$$

$$\therefore \Delta s = 0.718 \log_e \left(\frac{650}{300} \right) + 0.287 \log_e \left(\frac{1}{2.54} \right) +$$

$$= 0.555 + (-0.267) = 0.288 \text{ kJ/kgK}$$

$$\Delta s = 0.288 \text{ kJ/kgK}$$

Change in available energy

$$= m(h_1 - h_2 - T_0(S_1 - S_2))$$

$$= 8[0.718(650 - 300) - 300 \times 0.288] = 1319.2 \text{ kJ}$$

Loss of availability per unit mass during the process

$$= p_0(v_0 - v_1) \text{ per unit mass}$$

$$\text{Total loss of availability} = p_0(V_0 - V_1)$$

$$\text{But } V_1 = \frac{mRT_1}{p_1} = \frac{8 \times 287 \times 650}{5.5 \times 10^5}$$

$$= 2.713 \text{ m}^3$$

$$V_0 = 2.54 \times 2.713 = 6.891 \text{ m}^3$$

$$\therefore \text{Loss of availability} = \frac{1 \times 10^5}{10^3} (6.891 - 2.713) = 417.8 \text{ kJ. (Ans.)}$$

$$\text{Loss of availability} = 417.8 \text{ kJ. (Ans.)}$$

ii) Heat transferred during cooling (constant pressure) process

$$= m.c_p(T_1 - T_0)$$

$$= 8 \times 1.005 (650 - 300) = 2814 \text{ kJ}$$

Change in entropy during cooling

$$\Delta s = m c_p \log_e \frac{T_1}{T_0} = 8 \times 1.005 \times \log_e \left(\frac{650}{300} \right)$$

$$= 6.216 \text{ kJ/K}$$

$$\text{Unavailable energy} = T_0 \Delta S$$

$$= 300 \times 6.216 = 1864.8 \text{ kJ}$$

$$\text{Available energy} = 2814 - 1864.8 = 949.2 \text{ kJ. (Ans.)}$$

$$\text{Effectiveness, } \epsilon = \frac{\text{Available energy}}{\text{Change in Available energy}}$$

$$= \frac{949.2}{1319.2} = 0.719 \text{ (Ans.)}$$

Example4: What is the maximum useful work which can be obtained when 100 kJ

are abstracted from a heat reservoir at 675 K in an environment at 288 K? What is the loss of useful work if (a) A temperature drop of 50°C is introduced between the heat source and the heat engine, on the one hand, and the heat engine and the heat sink, on the other

(b) The source temperature drops by 50°C and the sink temperature rises by 50°C during the heat transfer process according to the linear

law $\frac{dQ}{dT} = \pm C \text{ constant?}$

$$Q = 100 \text{ kJ}$$

$$\text{heat reservoir } T_2 = 675 \text{ K environment } T_0 = 288 \text{ K}$$

Solution:

Entropy change for this process

$$\Delta S = \frac{Q}{T} = \frac{-100 \text{ kJ}}{675} = -0.14815 \text{ kJ/K}$$

(a) Now maximum work obtainable

$$W_{\max} = Q \left(1 - \frac{T_0 + 50}{T_2 - 50} \right)$$

$$W_{\max} = 100 \left(1 - \frac{338}{625} \right)$$

$$= 45.92 \text{ kJ}$$

$$\therefore \text{Loss of available work} = 57.333 - 45.92$$

$$= 11.413 \text{ KJ}$$

$$\text{b) Given : } \frac{dQ}{dT} = \pm C \text{ constant}$$

$$\text{Let } dQ = \pm m c_p dT$$

$\therefore$ When source temperature is (675 - T) and sink temperature (288 + T) at that time if dQ heat is flow then maximum.

Available work from that dQ is dW.

$$\begin{aligned} dW &= dQ \left(1 - \frac{288+T}{675-T} \right) \\ &= \left(1 - \frac{288+T}{675-T} \right) m c_p dT \end{aligned}$$

$$\begin{aligned} \therefore W_{\max} &= m c_p \int_0^{50} \left(1 - \frac{288+T}{675-T} \right) dT \\ &= m c_p \int_0^{50} \left(1 + 1 - \frac{288+T}{675-T} \right) dT \end{aligned}$$

$$=m c_p \{2(50-0) + 963 \ln\left(\frac{675-50}{675-0}\right)\}$$

$$=25.887 \text{ kJ}$$

$$m c_p \times 50 = 100 \text{ kJ}$$

$$=51.773 \text{ kJ}$$

$$\therefore m c_p = 2 \text{ kJ/K}$$

$$\therefore \text{Loss of availability} = (57.333 - 51.773) \text{ kJ}$$

$$=5.5603 \text{ kJ}$$

******Example 5:** Two kg of air at 500 kPa, 80°C expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which is at 100 kPa, 5°C. For this process, determine (a) the maximum work, (b) the change in availability, and (c) the irreversibility. For air, take $c_v = 0.718 \text{ kJ/kg K}$, $u = c_v T$ where c_v is constant, and $pV = mRT$ where p is pressure in kPa, V volume in m^3 , m mass in kg, R a constant equal to 0.287 kJ/kg K , and T temperature in K.

Solution:

$$\text{Pressure } p_1 = 500 \text{ kPa}$$

$$\text{Initial temperature, } T_1 = 80 + 273 = 353 \text{ K}$$

$$\text{Final temperature, } T_2 = 5 + 273 = 278 \text{ K}$$

$$\text{Surroundings Pressure } 100 \text{ kPa,}$$

$$\text{Surrounding temperature } 5^\circ\text{C,}$$

$$\text{To find: (a) the maximum work } W_{\max} = m(h_2 - h_1) - T_0(S_2 - S_1)$$

$$(b) \text{ the change in availability, } \phi_1 - \phi_2$$

$$(c) \text{ the irreversibility, } I = W_{\max} - W_{\text{act}}$$

$$\text{Calculate the maximum work } W_{\max}$$

$$= m(h_2 - h_1) + T_0(S_2 - S_1)$$

$$m = 2$$

$$(h_2 - h_1) = c_v(T_2 - T_1)$$

=

$$S_2 - S_1 = \Delta S = m \left[c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \right]$$

$$\Delta S = m \left(C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \right)$$

$$= (0.718 \ln \frac{278}{353} + 0.287 \ln \frac{2}{1}) = [(-0.172 + 0.199)]$$

$$= 7.51$$

$$S_2 - S_1 = \Delta S = 0.054$$

$$W_{max} = m(h_2 - h_1) + T_0(S_2 - S_1)$$

$$= 2 [c_v(T_1 - T_2)] + T_0(S_2 - S_1)$$

$$= 2(53.85 + 7.51)$$

$$= 122.72 \text{ kJ}$$

Calculate the change in availability

$$= \phi_1 - \phi_2 = m(h_2 - h_1) - T_0(S_2 - S_1) + p_0(V_0 - V_1)$$

$$= 122.72 \left[-100 \times \frac{2 \times 0.287 \times 353}{500} \right]$$

$$= 82.2 \text{ kJ}$$

Calculate the irreversibility. $I = W_{max} - W_{act}$

$$W_{act} = T_0 \Delta S = 278 \times 0.054 = 15.2 \text{ KJ}$$

Example 6 : A gas is flowing through a pipe at the rate of 2 kg/s. Because of inadequate insulation the gas temperature decreases from 800 to 790°C between two sections in the pipe. Neglecting pressure losses, calculate the irreversibility rate (or rate of energy degradation) due to this heat loss. Take $T_a = 300 \text{ K}$ and a constant $c_p = 1.1 \text{ kJ/kg K}$.

For the same temperature drop of 10°C when the gas cools from 80°C to 70°C due to heat loss, what is the rate of energy degradation? Take the same values of T_o and c_p . What is the inference you can draw from this

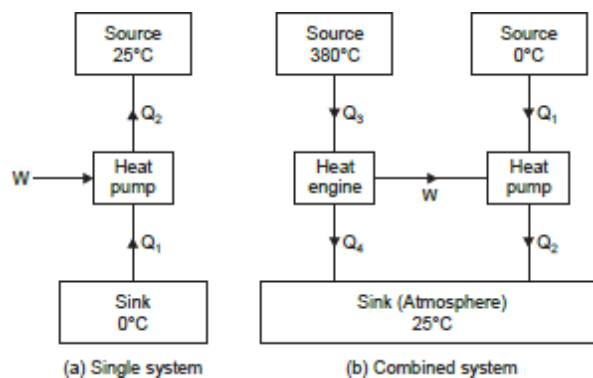
example?

Example: A reversible heat pump is used to maintain a temperature of 0°C in a refrigerator when it rejects the heat to the surroundings at 25°C . If the heat removal rate from the refrigerator is 1440 kJ/min , determine the C.O.P. of the machine and work input required. (ii) If the required input to run the pump is developed by a reversible engine which receives heat at 380°C and rejects heat to atmosphere, then determine the overall C.O.P. of the system.

Solution.

(i) Temperature, $T_1 = 25 + 273 = 298\text{ K}$

Temperature, $T_2 = 0 + 273 = 273\text{ K}$



Heat removal rate from the refrigerator,

$$Q_1 = 1440 \text{ kJ/min} = 24 \text{ kJ/s}$$

Now, co-efficient of performance, for reversible heat pump,

$$\text{C.O.P.} = \frac{T_1}{T_1 - T_2} = \frac{298}{(298 - 273)} = \mathbf{11.92. (Ans.)}$$

$$\therefore (\text{C.O.P.})_{\text{ref.}} = \frac{T_2}{T_1 - T_2} = \frac{273}{(298 - 273)} = 10.92$$

$$\text{Now, } 10.92 = \frac{Q_1}{W} = \frac{24}{W}$$

$$\therefore W = 2.2 \text{ kW}$$

i.e., **Work input required = 2.2 kW. (Ans.)**

$$Q_2 = Q_1 + W = 24 + 2.2 = 26.2 \text{ kJ/s}$$

(ii) Refer Fig. (b).

The overall C.O.P. is given by,

$$\text{C.O.P.} = \frac{\text{Heat removed from the refrigerator}}{\text{Heat supplied from the source}} = \frac{Q_1}{Q_3}$$

For the reversible engine, we can write

$$\frac{Q_3}{T_3} = \frac{Q_4}{T_4}$$

$$\frac{Q_4 + W}{653} = \frac{Q_4}{298}$$

$$298(Q_4 + 2.2) = 653Q_4$$

$$Q_4(653 - 298) = 298 \times 2.2$$

$$Q_4 = \frac{298 \times 2.2}{(653 - 298)} = 1.847 \text{ kJ/s}$$

$$\therefore Q_3 = Q_4 + W = 1.847 + 2.2 = 4.047 \text{ kJ/s}$$

Substituting this value in eqn. (i), we get

$$\text{C.O.P.} = \frac{24}{4.047} = 5.93. \text{ (Ans.)}$$

If the purpose of the system is to supply the heat to the sink at 25°C , then

$$\text{Overall C.O.P.} = \frac{Q_2 + Q_4}{Q_3} = \frac{26.2 + 1.87}{4.047} = 6.93. \text{ (Ans.)}$$

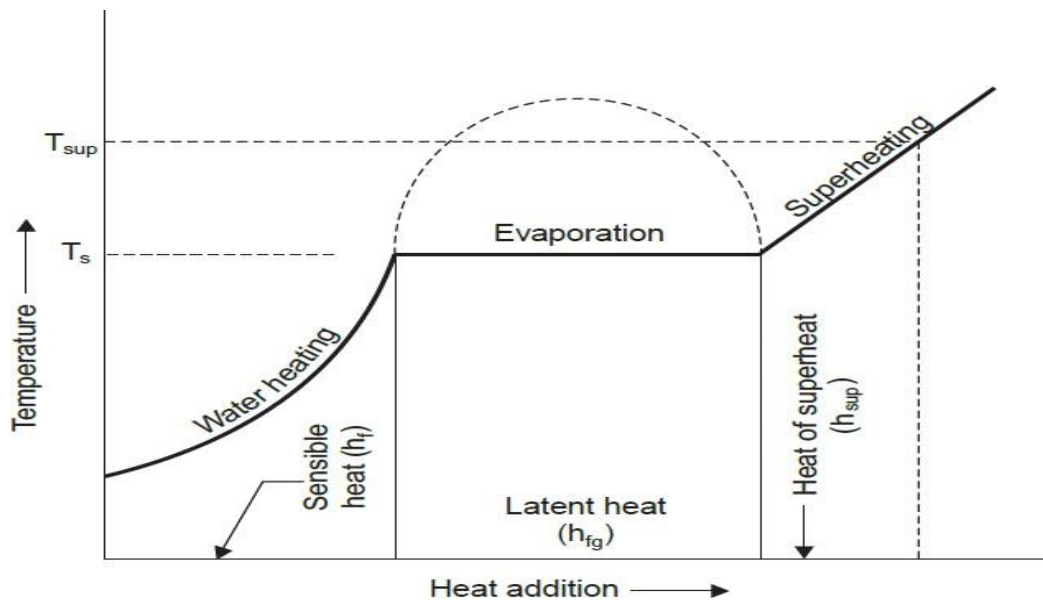
Unit-4

Introduction to Pure Substance

- There are three phases of substances: solid, liquid, and gases.
- Steam is a gaseous form of liquid.
- The vapourization of liquid is called vapour.
- The process of changing liquid to gaseous phase is called vapourization.

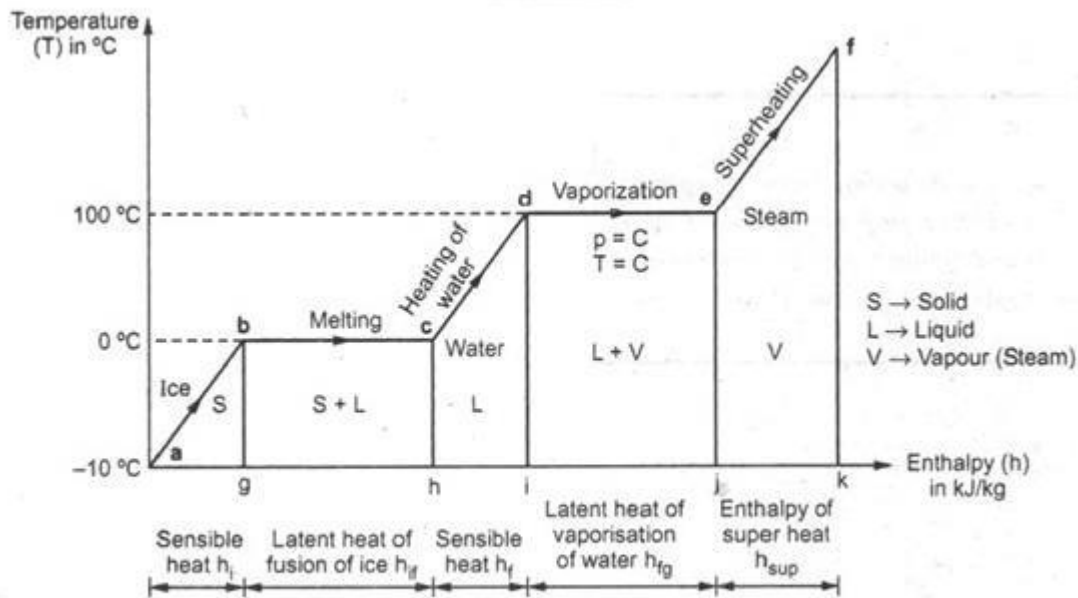
Phase Change at Constant Pressure

- Steam produced from water acts as a working fluid for a thermal power plant.
- Formation of steam from ice at constant pressure is shown in Fig. on a temperature-enthalpy ($T - h$) diagram.

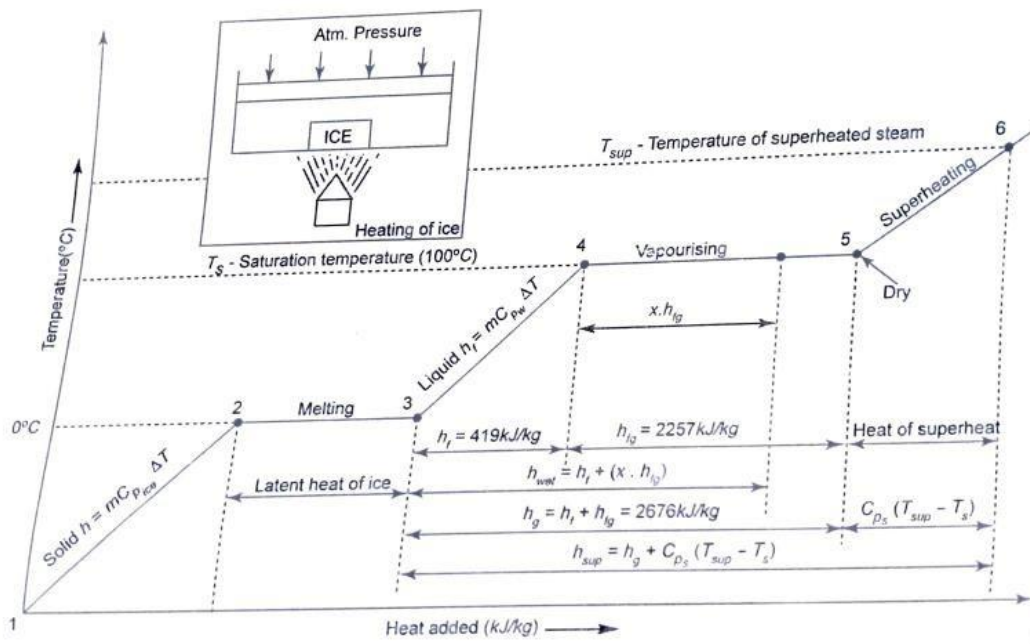


Process ab : As heat is supplied, temperature of ice increases from -15°C till it reaches 0°C . This is the sensible heating of ice

- Process bc: On further addition of heat, melting of ice takes place at constant temperature till the complete ice melts. The amount of heat added is latent heat of fusion or enthalpy of fusion.
- Process cd : Further heat addition after point C causes the increase in temperature of water upto 100°C . Water at point D is saturated liquid and this process of heating is sensible heating of water.
- Process de : This process shows the complete conversion of water to steam and amount of heat added is known as latent heat of vaporisation or enthalpy of vaporisation (h_{fg}). This heat is added at constant temperature.



. Graphical representation of formation of steam.

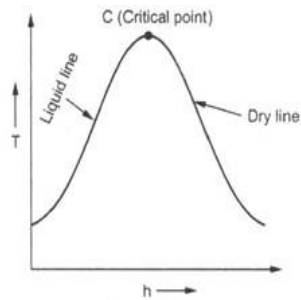


Process ef: This process represents the superheating of steam. The heat is being added to improve the quality of steam. The temperature of steam is raised above the 100 °C

Critical Point and Triple Point on Phase Diagram

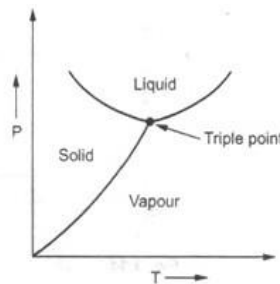
Critical Point:

- It is a point at which the latent heat becomes zero ($h_{fg}=0$).
- The following diagram shows critical point.
- In critical point the liquid is directly converted into dry steam



Triple Point:

- It is a point at which the solid, liquid, and gaseous phases meet.



Enthalpy of steam (h):

It is the amount of heat added to the water from freezing point to till the water becomes wet or dry or superheated steam.

For wet steam, $h_{wet} = h_f + x h_{fg}$

For dry steam, $h_{dry} = h_g = h_f + h_{fg}$

For superheated steam, $h_{sup} = h_g + C_p(T_{sup} - T_{sat})$

where,

h = Enthalpy of steam in **kJ/kg**

v = Specific volume of saturated steam

h_g = Enthalpy of dry saturated steam in **kJ/kg**

h_f = Enthalpy of water (4.187 kJ/kgK)

h_{fg} = Enthalpy of evaporation in kJ/kg

v_{sup} = Specific volume of superheated steam $T_s =$

Temperature of saturated steam

T_{sup} = Temperature of superheated steam

$(T_{sup} - T_{sat})$ is called as 'degree of superheat'.

h_{sup} = Enthalpy of superheated

steam C_p = Specific heat of superheated steam

am

= 2 kJ/kgK

(ii) Density of steam (ρ):

It is defined as the ratio of mass to the unit volume of steam at given pressure and temperature. Its value for wet, dry, and superheated steam is the reciprocal of specific volume of the steam.

(iii) Specific volume of steam (v):

It is defined as the volume occupied by the unit mass of steam at the given pressure and temperature.

It is denoted by V and is given by

$$v = \frac{1}{\rho}$$

where,

v = Specific volume ρ = Density

For wet steam, $v_{wet} = x v_g = (1 - x) v_f + x v_g$ For

dry steam, $= v_g$

For superheated steam, $v_{sup} = \frac{v_g T_{sup}}{T_{sat}}$

where

v = Specific volume in m^3/kg

(iv) Work done during expansion (W):

energy required for absorption of latent heat for increasing volume of the steam.

$$\text{For wet steam, } W_{\text{wet}} = 100 p x v_g$$

$$\text{For dry steam, } W_{\text{dry}} = 100 p v_g$$

$$\text{For superheated steam, } W_{\text{sup}} = 100 p v_{\text{sup}}$$

where,

$$W = \text{Work done in kJ}$$

$$p = \text{Pressure at which evaporation takes place in bar. } (v) \text{ Internal}$$

energy of steam (U):

Internal energy of steam is defined as the actual heat energy stored in the steam above the freezing point of water at the given conditions. It is the difference between enthalpy of steam and the external work done.

$$h = u + W_e = u + p v$$

$$W_e = p v = \text{Evaporation work}$$

$$\text{For wet steam, } u_{\text{wet}} = [h_f + x h_{fg}] - [100 p x v_g] \text{ For}$$

$$\text{dry steam, } U_{\text{dry}} = [h_f + h_{fg}] - [100 p v_g]$$

$$\text{For superheated steam, } u_{\text{sup}} = h_{\text{sup}} - [100 p v_{\text{sup}}]$$

where

$$u = \text{Enthalpy of steam in kJ/kg}$$

(iv) Entropy of steam (s):

It is the property of the steam which increases with increase in temperature and decreases with decrease in temperature.

$$\text{For wet steam, } s_{wet} = s_f + x s_{fg}$$

$$\text{For dry steam } s_{dry} = s_f + s_{fg}$$

$$\text{For superheated steam, } s_{sup} = s_g + C_{ps} \log \left(\frac{T_{sup}}{T_s} \right)$$

where

$$s = \text{Entropy of steam in kJ/kgK}$$

The above mentioned properties at different pressures are tabulated in the form of tables as

p = Absolute pressure (bar or kPa);

s_f = Entropy of saturated liquid (kJ/kgK);

s_{fg} = Entropy of vapourisation (kJ/kgK);

s_g = Entropy of saturated vapour (steam) (kJ/kgK);

v_f = Specific volume of saturated liquid (m^3/kg);

v_g = Specific volume of saturated vapour (steam) (m^3/kg).

Change of enthalpy during evaporation,

$$h_{fg} = h_g - h_f \dots$$

Change of entropy during evaporation

$$s_{fg} = s_g - s_f \dots$$

Change of volume during evaporation.

$$v_{fg} = v_g - v_f \dots$$

Internal energy of steam

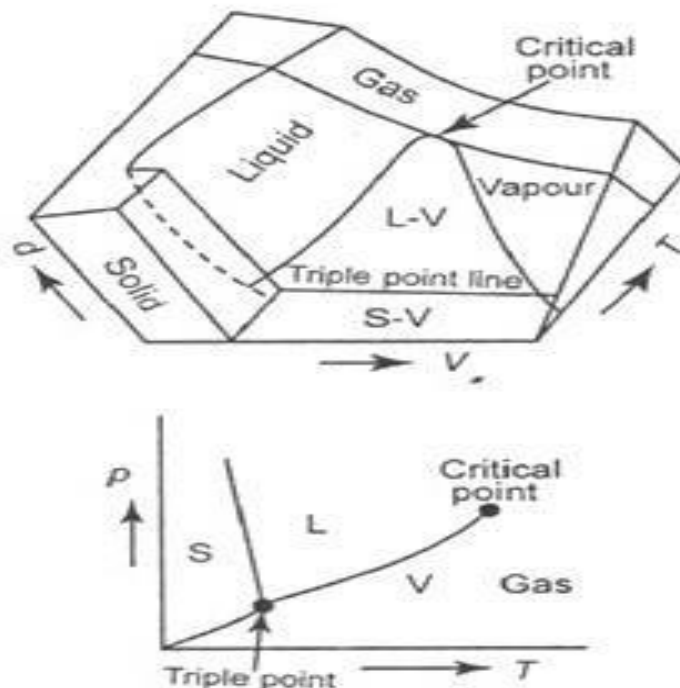
$$u = h - pv$$

External work of evaporation

$$= p(v_g - v_f)$$

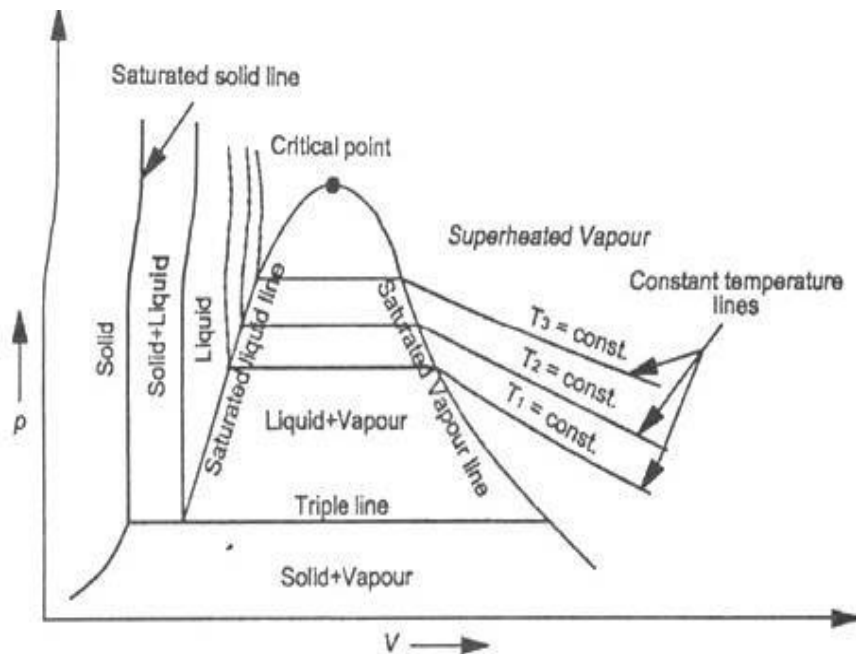
p-v-T SURFACE

- The three important thermodynamic properties such as pressure (p), specific volume (v) and temperature (T) are plotted in three dimensional coordinates. This is called as **p - v - T** surface.
- That is represented on both the T - V and P - T diagram can be shown on one diagram if the three coordinates P , V and T are plotted along the rectangular axes.
- The obtained surface is called as the PVT surface as shown in figure
- When the surface is projected on the PT plane, the solid-vapour region projects into the sublimation curve, the liquid-vapour region projects into the fusion curve and the triple point line projects into the triple point.
- The critical point is denoted by the letter C and the triple point by T

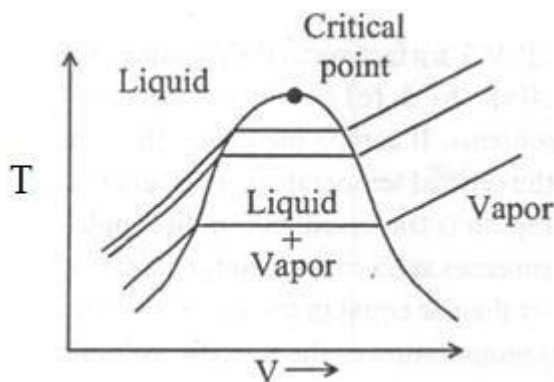


p - V DIAGRAM

p - V diagram is the plot between specific volume and pressure. Specific volume is taken along X-axis and pressure is taken along X-axis. The state changes of a pure substance when it is slowly heated at different constant pressures as shown in Figure Most of the substances contract during a solidification or freezing process. Some substances such as water expand as they freeze. **p - V** diagram type of substances are given in the Figure



The liquid, two phase liquid-vapour and vapour regions of the P-V-T surface projected on the Temperature- volume plane results in a T-V diagram, as shown in Figure. The T-V diagram is also convenient for solving problems. For pressure less than the critical pressure, the pressure remains constant with temperature in the two phase flow region. In the single phase liquid and vapour regions, the pressure decreases as the specific volume increases for pressures greater than or equal to the critical pressure.



Example1 : Calculate the dryness fraction (quality) of steam which has 1.5 kg of water in suspension with 50 kg of steam.

Solution. Mass of dry steam, $m_g = 50$ kg

Mass of water in suspension, $m_f = 1.5$ kg

∴ Dryness fraction, $x = \frac{m_g}{m_f + m_g}$

Dryness fraction, $\frac{\text{Mass of dry steam}}{\text{Mass of dry steam} + \text{mass of water in suspension}}$
 $\frac{50}{50 + 1.5} = \mathbf{0.971. (Ans.)}$

Example 2 : Ten kg of water at 45°C is heated at a constant pressure of 10 bar until it becomes superheated vapour at 300°C . Find the change in volume, enthalpy, internal energy and entropy.

Given:

Mass $m = 10$ kg at 45°C

Pressure $p = 10$ bar.

Superheated vapour at 300°C .

From Steam table: $p = 10$ bar.

To find;

i) change in volume,

ii) enthalpy,

iii) internal energy

iv) entropy.

At state (1)	At state (2)
$p_1 = 10 \text{ bar} = 1000 \text{ kPa}$	$p_2 = p_1 = 10 \text{ bar}$
$T_1 = 45^\circ\text{C} = 318 \text{ K}$	$T_2 = 300^\circ\text{C}$

$$v_1 = 0.001010 \text{ m}^3/\text{kg} \quad v_2 = 0.258 \text{ m}^3/\text{kg}$$

$$h_1 = 188.4 \text{ kJ/kg} \quad u_2 = 2793.2 \text{ kJ/kg}$$

$$u_1 = h_1 - p_1 v_1 = 187.39 \text{ kJ/kg} \quad h_2 = 3051.2 \text{ kJ/kg} \quad s_1 =$$

$$0.693 \text{ kJ/kg} - \text{K} \quad s_2 = 7.123 \text{ kJ/kg} - \text{K}$$

$$\therefore \text{Change in volume} = m(v_2 - v_1) = 2.57 \text{ m}^3$$

$$\text{Enthalpy change} = m(h_2 - h_1) = 28.628 \text{ MJ}$$

$$\text{Internal Energy change} = m(u_2 - u_1) = 26.0581 \text{ MJ}$$

$$\text{Entropy change} = m(s_2 - s_1) = 64.3 \text{ kJ/K}$$

Example 3: A rigid vessel of volume 0.86 m^3 contains 1 kg of steam at a pressure of 2 bar . Evaluate the **specific volume, temperature, dryness fraction, internal energy, enthalpy, and entropy of steam.**

Given: volume $v = 0.86 \text{ m}^3$ Mass
 $m = 1 \text{ kg}$

Pressure $p = 2 \text{ bar}$.

From Steam table: $p = 2 \text{ bar}$.

t_s	v_f	V_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
120.2	0.001061	0.885	504.7	2201.6	2706.3	1.530	5.597	7.127

Calculate the specific volume

$$\text{Specific volume} = \frac{\text{Volume}}{\text{mass}} = \frac{0.86}{1} = 0.86 \text{ m}^3/\text{kg}$$

Calculate the dryness fraction

$$v = v_f + x(v_g - v_f)$$

$$\therefore \text{dryness fraction } x = \frac{v - v_f}{v_g - v_f} = \frac{0.86 - 0.001061}{0.885 - 0.001061} = 0.97172$$

Calculate the Internal energy (u)

$$u = h - pv = 2644 - 200 \times 0.86 = 2472 \text{ kJ/kg}$$

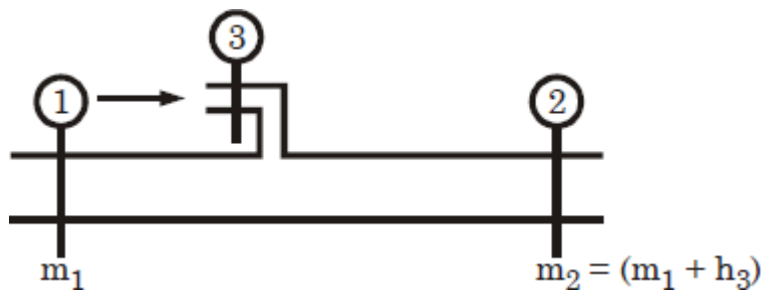
Calculate the enthalpy,

$$h = h_f + x h_{fg} = 504.7 + 0.97172 \times 2201.6 = 2644 \text{ kJ/kg}$$

Calculate the entropy of steam.

$$s = s_f + x s_{fg} = 1.5301 + 0.97172 \times 5.5967 = 6.9685 \text{ kJ/kg} - \text{K}$$

Example 4 : Water at 40°C is continuously sprayed into a pipeline carrying 5 tonnes of steam at 5 bar , 300°C per hour. At a section downstream where the pressure is 3 bar , the quality is to be 95% . Find the rate of water spray in kg/h .



$P_1 = 5 \text{ bar} = 500 \text{ kPa}$	$P_2 = 3 \text{ bar}$
$T_2 = 300^\circ\text{C}$	$T_2 = 133.5^\circ\text{C}$
$h_1 = 3064.2 \text{ kJ/kg}$	$h_2 = 561.4 + 0.95 \times 2163.2$ $= 2616.44 \text{ kJ/kg}$

$$T_3 = 40^\circ\text{C}$$

$$h_3 = 167.6 \text{ kJ/kg}$$

∴ **For adiabatic steady flow**

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_3) h_2$$

$$\dot{m}_1 (h_1 - h_2) = \dot{m}_3 (h_2 - h_3)$$

$$\begin{aligned} \dot{m}_3 &= \dot{m}_1 \frac{(h_1 - h_2)}{(h_2 - h_3)} \\ &= 5000 \times \frac{3064.2 - 2616.44}{2616.44 - 167.6} \text{ kg/hr} \\ &= 914.23 \text{ kg/hr.} \end{aligned}$$

Example 5: A vessel having a volume of 0.6 m^3 contains 3.0 kg of liquid water and water vapour mixture in equilibrium at a pressure of 0.5 MPa . Calculate :

- (i) Mass and volume of liquid ;
- (ii) Mass and volume of vapour.

Solution.

$$\text{Volume of the vessel, } V = 0.6 \text{ m}^3$$

$$\text{Mass of liquid water and water vapour, } m = 3.0 \text{ kg}$$

$$\text{Pressure, } p = 0.5 \text{ MPa} = 5 \text{ bar}$$

$$\text{specific volume, } v = \frac{V}{m} = \frac{0.6}{3.0} = 0.2 \text{ m}^3/\text{kg}$$

At 5 bar: From steam tables,

t_s	v_f	V_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
151.8	0.001093	0.3746	640.1	2107.4	2747.5	1.860	4.959	6.819

Calculate the dry fraction

$$v_{fg} = v_g - v_f = 0.375 - 0.00109 = 0.3739 \text{ m}^3/\text{kg} \text{ We}$$

know that,

$$v = v_g - (1-x)v_{fg}, \text{ where } x = \text{quality of the vapour.}$$

$$0.2 = 0.375 - (1-x) \times 0.3739$$

$$\therefore (1-x) = \frac{(0.375 - 0.2)}{0.3739} = 0.468$$

$$x = 0.532$$

(i) Mass and volume of liquid, $m_{liq} = ?$ $V_{liq} = ?$

$$m_{liq} = m(1 - x) = 3.0 \times 0.468 = 1.404 \text{ kg. (Ans.)}$$

$$V_{liq} = m_{liq} \cdot v_f = 1.404 \times 0.00109 = 0.0015 \text{ m}^3. \text{ (Ans.)}$$

(ii) Mass and volume of vapour, $m_{vap} = ?$ $V_{vap} = ?$

$$m_{vap} = m \cdot x = 3.0 \times 0.532 = 1.596 \text{ kg. (Ans.)}$$

$$V_{vap} = m_{vap} \cdot v_g = 1.596 \times 0.375 = 0.5985 \text{ m}^3. \text{ (Ans.)}$$

Example 6 : 1000 kg of steam at a pressure of 16 bar and 0.9 dry is generated by a boiler per hour. The steam passes through a superheater via boiler stop valve where its temperature is raised to 380°C. If the temperature of feed water is 30°C, determine :

(i) The total heat supplied to feed water per hour to produce wet steam.

(ii) The total heat absorbed per hour in the superheater.

Take specific heat for superheated steam as 2.2 kJ/kgK.

Solution.

Mass of steam generated, $m = 1000 \text{ kg/h}$

Pressure of steam, $p = 16 \text{ bar}$

Dryness fraction, $x = 0.9$

Temperature of superheated steam,

$$T_{sup}=380+273=653K$$

Temperature of feed water = 30°C

Specific heat of superheated steam, $c_{ps}=2.2\text{kJ/kgK}$.

At 16 bar. From steam tables,

$$t_s=201.4^\circ\text{C} (T_s=201.4+273=474.4\text{K});$$

$$h_f=858.6\text{kJ/kg}; h_{fg}=1933.2\text{ kJ/kg}$$

(i) Heat supplied to feed water per hour to produce wet steam is given by
:

$$\begin{aligned} H &= m[(h_f + x h_{fg}) - 1 \times 4.18 \times (30 - 0)] \\ &= 1000[(858.6 + 0.9 \times 1933.2) - 4.18 \times 30] \\ &= 1000(858.6 + 1739.88 - 125.4) \\ &= \mathbf{2473.08 \times 103 \text{ kJ. (Ans.)}} \end{aligned}$$

$$\begin{aligned} \text{(ii) Heat absorbed by superheater per hour} \\ &= m[(1-x)h_{fg} + c_{ps}(T_{sup} - T_s)] \\ &= 1000[(1-0.9) \times 1933.2 + 2.2(653 - 474.4)] \\ &= 1000(193.32 + 392.92) \\ &= \mathbf{586.24 \times 103 \text{ kJ. (Ans.)}} \end{aligned}$$

*****Example 7 :** A vessel having a capacity of 0.05 m³ contains a mixture of saturated water and saturated steam at a temperature of 245°C. The mass of the liquid present is 10 kg. Find the following:

- (i) The pressure, (ii) The mass,
(iii) The specific volume, (iv) The specific enthalpy,
(v) The specific entropy, and (vi) The specific internal energy.

Solution.

From steam tables, corresponding to 245°C:

p_{sat}	v_f	V_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
36.5 bar	0.001239	0.0546	1061.4	1740.2		2.7474		3.3585

i) The pressure = 36.5 bar

(ii) The mass, m :

Volume of liquid,

$$V_f = m_f v_f [\text{mass} \times \text{Specific volume of saturated}]$$

$$=10 \times 0.001239 = 0.01239 \text{ m}^3$$

Total volume of the vessel

$$V = \text{volume of liquid} + \text{volume of steam} \\ = V_1 + V_2$$

$$0.05 = 0.01239 + V_g$$

Volume of vapour, V_g

$$= 0.05 - 0.01239 = 0.03761 \text{ m}^3$$

$$\therefore \text{Mass of vapour, } m_g = \frac{V_g \rho_g}{V_g} = \frac{0.03761}{0.0546}$$

$$= 0.688 \text{ kg}$$

$\therefore$ The total mass of mixture,

$$m = m_f + m_g = 10 + 0.688 = \mathbf{10.688 \text{ kg. (Ans.)}}$$

(iii) Calculate the specific volume, v :

$$\text{Quality of the mixture } x = \frac{m_g}{m_g + m_f} = \frac{0.688}{0.688 + 10} = 0.064$$

.

$$\therefore v = v_f + x v_{fg}$$

$$= 0.001239 + 0.064 \times (0.0546 - 0.001239) \quad (\because v_{fg} = v_g - v_f)$$

$$= \mathbf{0.004654 \text{ m}^3/\text{kg. (Ans.)}}$$

(iv) The specific enthalpy, h :

$$h = h_f + x h_{fg}$$

$$= 1061.4 + 0.064 \times 1740.2 = \mathbf{1172.77 \text{ kJ/kg. (Ans.)}}$$

(v) The specific entropy, s :

$$s = s_f + x s_{fg}$$

$$= 2.7474 + 0.064 \times 3.3585 = \mathbf{2.9623 \text{ kJ/kgK. (Ans.)}}$$

(vi) The specific internal energy, u :

$$u = h - p v$$

$$= 1172.77 - \frac{36.5 \times 0.004654}{1000} = \mathbf{1155.78 \text{ kJ/kg. (Ans.)}}$$

 **Example 3.9.** A pressure cooker contains 1.5 kg of saturated steam at 5 bar. Find the quantity of heat which must be rejected so as to reduce the quality to 60% dry. Determine the pressure and temperature of the steam at the new state.

Solution.

Mass of steam in the cooker = 1.5 kg Pressure of steam, $p = 5 \text{ bar}$

Initial dryness fraction of steam, $x_1 = 1$ (saturated steam)

Final dryness fraction of steam, $x_2 = 0.6$ (dry)

Heat to be rejected:

Pressure and temperature of the steam at the new state: At 5 bar. From steam tables,

$$t_s = 151.8^\circ\text{C}; h_f = 640.1 \text{ kJ/kg};$$

$$h_{fg} = 2107.4 \text{ kJ/kg}; v_g = 0.375 \text{ m}^3/\text{kg} \text{ volume of pressure cooker}$$

$$= m \times v_g \\ = 1.5 \times 0.375 = 0.5625 \text{ m}^3$$

Internal energy of steam per kg at initial point 1,

$$u_1 = h_1 - p_1 v_1$$

$$= (h_f + h_{fg}) - p_1 v_{g1} \quad (\text{ } v_1 = v_{g1})$$

$$= (640.1 + 2107.4) - 5 \times 10^5 \times 0.375 \times 10^{-3}$$

$$= 2747.5 - 187.5 = 2560 \text{ kJ/kg}$$

Also, $V_1 = V_2$ ($V_2 = \text{volume at final condition}$)

$$\text{i.e., } 0.5625 = 1.5[(1 - x_2)v_{f2} + x_2 v_{g2}]$$

$$= 1.5 x_2 v_{g2} \quad (\text{ } v_{f2} \text{ is negligible})$$

$$= 1.5 \times 0.6 \times v_{g2}$$

$$\therefore v_{g2} = \frac{0.5625}{1.5 \times 0.6} = 0.625 \text{ m}^3/\text{kg}.$$

From steam tables corresponding to $0.625 \text{ m}^3/\text{kg}$,

$p_2 = 2.9 \text{ bar}$, $t_s = 132.4^\circ\text{C}$, $h_f = 556.5 \text{ kJ/kg}$, $h_{fg} = 2166.6 \text{ kJ/kg}$ Internal energy of steam per kg, at final point 2,

$$u_2 = h_2 - p_2 v_2$$

$$= (h_f + x_2 h_{fg}) - p_2 x v_{g2}$$

$$(\text{ } v_2 = x v_{g2})$$

$$= (556.5 + 0.6 \times 2166.6) - 2.9 \times 10^5 \times 0.6 \times 0.625 \times 10^{-3}$$

$$= 1856.46 - 108.75 = 1747.71 \text{ kJ/kg}.$$

Heat transferred at constant volume per kg

$$= u_2 - u_1 = 1747.71 - 2560 = -812.29 \text{ kJ/kg}$$

Thus, **total heat transferred**

$$= -812.29 \times 1.5 = -1218.43 \text{ kJ. (Ans.)}$$

Negative sign indicates that the heat has been rejected.

Maxwell's equation

Maxwell's relations are a set of equations in thermodynamics which are derivable from the symmetry of second derivatives and from the definitions of the thermodynamic potentials.

Thermodynamic relation was developed by single phase having only 2 independent variables and four thermodynamic properties.

- 1) Internal energy (U)
- 2) Enthalpy ($H = U + pV$)
- 3) Helmholtz free energy ($F = U - TS$)
- 4) Gibbs free energy ($G = H - TS$)

According to first law

$$dQ = du + p dv \quad \dots (1)$$

$$\text{second law, } ds = \frac{dQ}{T}$$

$$dQ = T ds$$

Substitute value of dQ in eqn: 1

$$T ds = du + p dv$$

$$du = T ds - p dv \dots (2)$$

For Enthalpy $H = U + pV$

Differential change in internal energy equation

$$dh = du + v dp + p dv \dots (3)$$

Substitute du value of eqn 2

$$T ds - p dv + v dp + p dv$$

$$dh = T ds + v dp \dots (4)$$

Helmholtz free energy function, $F = U - TS$

$$df = du - T ds - s dT \dots (5)$$

Substitute du value of eqn 5

$$df = Tds - pdv - Tds - sdT \dots$$

$$= -pdv - sdT \dots (6)$$

Gibb's free energy function,

$$G = H - TS,$$

$$dg = dh - Tds - sdT \dots (7)$$

Substituted h value of in 7

$$\begin{aligned} dg &= -Tds + vdp + Tds - sdT \\ &= vdp - sdT \dots (8) \end{aligned}$$

$$du = Tds - pdv$$

$$dh = Tds + vdp$$

$$df = -pdv - sdT$$

$$dg = vdp - sdT$$

thermodynamic properties must satisfy the theorem-I

theorem-II given by

$$\left(\frac{\partial M}{\partial y_x} \right) = \left(\frac{\partial N}{\partial x_y} \right)$$

$$du = Tds - pdv \quad \text{become}$$

$$\left(\frac{\partial T}{\partial v_s} \right) = - \left(\frac{\partial P}{\partial s_v} \right)$$

$$dh = Tds + vdp \quad \text{become}$$

$$\left(\frac{\partial T}{\partial P_s} \right) = \left(\frac{\partial v}{\partial P_s} \right)$$

$$df = -pdv - sdT \quad \text{become}$$

$$\left(\frac{\partial P}{\partial T_v} \right) = \left(\frac{\partial s}{\partial T_v} \right)$$

$$dg = vdp - sdT \quad \text{become}$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial S}{\partial P}\right)_T$$

These are known as **Maxwell's equations**

ENTROPY RELATIONS

Tds Equation

Entropy (s) may be expressed as a function of any other two properties, e.g. temperature (T) and pressure (p)

$$S = f(T, p)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

$$\text{Where } C_p = T \left(\frac{\partial s}{\partial T}\right)_p : \left(\frac{\partial s}{\partial T}\right)_p = \frac{C_p}{T}$$

$$\text{From Maxwell relation, } \left(\frac{\partial s}{\partial p}\right)_T = - \left(\frac{\partial v}{\partial T}\right)_p$$

Substituting the above expression into the equation

$$ds = \frac{C_p}{T} dT - \left(\frac{\partial v}{\partial T}\right)_p dp$$

Multiplying by T on both sides of the equation

$$T ds = C_p dT - T \left(\frac{\partial v}{\partial T}\right)_p dp$$

This is known as the first form of Entropy equation or first Tds equation.

Entropy (s) may be expressed as a function of any other two properties, e.g. temperature T and specific volume v,

$$S = f(T, v)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

$$\text{Where } C_v = T \left(\frac{\partial s}{\partial T_v} \right) : \left(\frac{\partial s}{\partial T_v} \right) = \frac{C_v}{T}$$

$$\text{From Maxwell relation, } \left(\frac{\partial P}{\partial T_v} \right) = \left(\frac{\partial S}{\partial V_T} \right)$$

Substituting the above expression in the above Equation

$$ds = \frac{C_v}{T} dt + \left(\frac{\partial P}{\partial T_v} \right) dv$$

Multiplying by T

$$T ds = C_v dT + T \left(\frac{\partial P}{\partial T_v} \right) dv$$

This is known as the second form of Entropy equation or second Tds equation.

Joule-Thomson Coefficient (p).

The *slope* of the constant enthalpy curve on a T - p diagram at any state point is called **Joule - Thomson Coefficient** (p). It is defined as the ratio of change of temperature : change in pressure at any state under constant enthalpy condition .

$$\mu_j = \left(\frac{dT}{dp_h} \right)$$

EXPRESSION FOR JOULE-THOMSON CO-EFFICIENT

Value of μ_{JT} is mathematically represented as

$$\mu_{JT} = \left(\frac{\partial T}{\partial p_h} \right)$$

We know that

$$dh = T ds + v dp \quad (H = U + pv) \quad \text{Substituting for}$$

TdS

The second TdS equation is

$$TdS = mc_p dT - T \left(\frac{\partial v}{\partial T} \right)_p dp$$

Equation becomes $dh = mc_p dT - \left(T \left(\frac{\partial v}{\partial T} \right)_p - v \right) dp + v dp$

$$= mc_p dT - \left(T \left(\frac{\partial v}{\partial T} \right)_p - v \right) dp$$

For unit mass, the above equation becomes

$$= c_p dT - \left(T \left(\frac{\partial v}{\partial T} \right)_p - v \right) dp$$

$$= c_p (dT)_h - \left(T \left(\frac{\partial v}{\partial T} \right)_p - v \right) (dp)_h \quad \therefore \text{if } h = C \text{ then } dh = 0$$

$$\left(\frac{dT}{dp} \right)_h = \frac{1}{c_p} \left(T \left(\frac{\partial v}{\partial T} \right)_p - v \right)$$

$$\mu_j = \frac{1}{c_p} \left(T \left(\frac{\partial v}{\partial T} \right)_p - v \right)$$

$$\mu_j = \frac{1}{c_p} \left(T (\beta v)_p - v \right) \quad \left(\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p \right)$$

$$\mu_j = \frac{v}{c_p} (T (\beta)_p - 1)$$

CLAUSIUS-CLAPEYRON EQUATION

Clausius-Clapeyron equation provides a relationship between various properties during a phase change process.

First TdS equation is given by

$$TdS = mc_v dT + T \left(\frac{\partial p}{\partial T} \right)_v dv$$

For any phase change process, the temperature & pressure remains constant

Therefore $dt = 0$

$$TdS = T \left(\frac{\partial p}{\partial T_v} \right) dv \text{ (or) } ds = T \left(\frac{\partial p}{\partial T_v} \right) dv$$

Integrating between initial state '1' & final state '2'

$$\int_1^2 ds = \int_1^2 \left(\frac{\partial p}{\partial T_v} \right) dv$$

$$s_2 - s_1 = \left(\frac{dp}{dT_v} \right) \int_1^2 dv$$

$$s_2 - s_1 = \left(\frac{dp}{dT_v} \right) (v_2 - v_1)$$

$$\frac{dp}{dT} = \frac{s_2 - s_1}{v_2 - v_1}$$

From second law of thermodynamics,

$$dh = Tds + vdp \text{ At}$$

stated earlier, as $P = \text{const}$, $dp = 0$

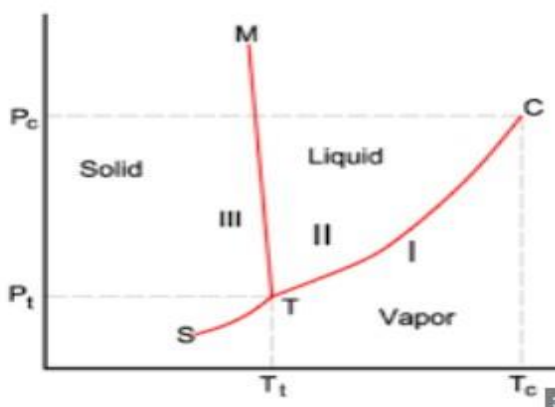
$$dh = Tds$$

Integrating between initial state '1' & '2'

$$\int_1^2 dh = \int_1^2 Tds$$

$$h_2 - h_1 = T(s_2 - s_1)$$

$$(s_2 - s_1) = \frac{(h_2 - h_1)}{T}$$



$$\frac{dp}{dT} = \frac{(h_2 - h_1)}{T(s_2 - s_1)}$$

The above equation is known as Clausius–Clapeyron equation

Clausius–Clapeyron for

open & closed system, $ds = ds_{\text{sys}} + ds_{\text{surr}}$

Isolated system $ds \geq 0$

Phase change process. Process that occurs that a substance changes its state from one of three states, liquid & vapour to another

