M. Dharaseelan
MECH-II

Reg. No. : E N G G T R E E . C O M

K. Manibarathi

Question Paper Code : 70138

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Mechanical Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aeronautical Engineering/Aerospace Engineering/
Automobile Engineering/Biomedical Engineering/
Civil Engineering/Manufacturing Engineering/Marine Engineering/
Materials Science and Engineering/Mechanical Engineering (Sandwich)/
Mechanical and Automation Engineering/Mechatronics Engineering/
Medical Electronics/ Petrochemical Engineering/Production Engineering/
Robotics and Automation/Safety and Fire Engineering/Bio Technology/
Biotechnology and Biochemical Engineering/Food Technology/
Petrochemical Technology/ Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the complete solution of the PDE $p^2 + q^2 = 4pq$.

2. Find the complementary function of the PDE $(D^3 - 3DD'^2 + 2D'^3)z = e^{2x-y}$.

3. Identify the given $f(x)$ is an even or odd function. Also sketch its graph.

$$f(x) = \begin{cases} \pi + x & -\pi \le x \le -\frac{\pi}{2} \\ -x & -\frac{\pi}{2} \le x \le 0 \\ x & 0 \le x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x \le \pi \end{cases}$$

4. State Parseval's identity in Fourier series.

5. Write down the appropriate solution of the one dimensional heat flow equation. How is it chosen?

6. The ends A and B of a rod 30 cm long, have their temperature kept at 10°C and 100°C respectively. Then obtain the steady state temperature.

7. What are the sufficient conditions for the existence of Fourier transform of a function $f(x)$?

8. Obtain the Fourier cosine transform of $\frac{1}{2^t}$.

9. Find the inverse Z transform of $\frac{z}{(z-1)^2}$.

10. State final value theorem in Z transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $(x-2z)p+(2z-y)q = y-x$. (8)

(ii) Solve : $(D^3+D^2D'-4DD'^2-4D'^3)z = \cos(2x+y)$. (8)

Or

(b) (i) Solve the PDE $2z+p^2+qy+2y^2=0$. (8)

(ii) Solve $(D^2+2DD'+D'^2-2D-2D')z=\sin(x+2y)$. (8)

12. (a) (i) Obtain the Fourier series of periodicity 2π for $f(x)=e^x$ in the interval $0<x<2\pi$. (8)

(ii) Obtain the half range Fourier cosine series of $f(x)=x(l-x)$ in $(0, l)$. (8)

Or

(b) The following table gives the variations of periodic current over a period.

t sec :	0	T/6	T/3	T/2	2T/3	5T/6	T
A amp :	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. (Harmonic Analysis). (16)

13. (a) A tightly stretched string of length $2l$ is fastened at both ends. The midpoint of the string is displaced by a distance 'b' transversely and the string is released from rest in this position. Find the displacement y at any distance x from one end at any time t. (16)

Or

(b) An infinitely long metal plate in the form of an area is enclosed between the lines $y = 0$ and $y = \pi$ for positive values of x. The temperature is zero along the edges $y = 0$ and $y = \pi$ and the edge at infinity. If the edge $x = 0$ is kept at temperature 'ky', find the steady state temperature at any point in the plate. (16)

14. (a) (i) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1, & for\ |x| \le a \\ 0, & for\ |x| > a \end{cases}$. (8)

(ii) Find the Fourier sine transform of $f(x) = e^{-ax}$, $a > 0$ and hence find $F_C(xe^{-ax})$. (8)

Or

(b) (i) Using Parseval's identity for Fourier transforms, evaluate $\int_0^\infty \frac{ds}{(a^2+s^2)(b^2+s^2)}$. (8)

(ii) Find the Fourier cosine transform of $f(x) = \begin{cases} x & ;\ 0 < x < 1 \\ 2-x & ;\ 1 < x < 2 \\ 0 & ;\ x > 2 \end{cases}$. (8)

15. (a) Using Z transform, solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0$, $u_1 = 1$. (16)

Or

(b) State and prove convolution theorem in Z transforms and use it to find $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$. (16)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 30240

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third Semester

Aeronautical Engineering

MA 3351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aerospace Engineering/Automobile Engineering/Biomedical Engineering/Civil Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/ Mechanical Engineering (Sandwich)/ Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation/Safety and Fire Engineering/Bio Technology/Biotechnology and Biochemical Engineering/Food Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the PDE by eliminating the arbitrary constants 'a and b' from the relation $z = (x + a)(y + b)$.

2. Find the PDE corresponding to the complementary function $z = f_1(y + x) + x f_2(y + x) + f_3(y - x) + x f_4(y - x)$.

3. If $f(x)$ is defined in $(-\pi, \pi)$ and if $f(x) = x + 1$ in $(0, \pi)$, then find $f(x)$ in $(-\pi, 0)$ if

   (a) $f(x)$ is odd

   (b) $f(x)$ is even.

4. Determine the value of b_{25} while expanding the function $f(x) = \begin{cases} 1 + \frac{2x}{l}; & -l \le x \le 0 \\ 1 - \frac{2x}{l}; & 0 \le x \le l \end{cases}$ as a Fourier series.

5. A tightly stretched string of length '$2L$' is fastened at both ends. The midpoint of the string is displaced to a distance 'b' and released from rest in this position. Write the boundary conditions.

6. The ends A and B of a rod 100 cm long, have their temperatures kept at 10°C and 100°C respectively. Then find the steady state temperature distribution function.

7. Obtain the Fourier transform of $f(x) = \begin{cases} 1; & for\ |x| \le 1 \\ 0; & for\ |x| > 1. \end{cases}$

8. State Convolution theorem for Fourier transforms.

9. Find the inverse Z transform of the unit impulse sequence $\delta(n) = \begin{cases} 1; & for\ n = 0 \\ 0; & for\ n \ne 0. \end{cases}$

10. State Initial value theorem in Z transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve : $(x-2z)p + (2z-y)q = y-x$ (8)

(ii) Solve : $(D^2 + DD' - 6D'^2)z = \cos(2x+y)$. (8)

Or

(b) (i) Solve : $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. (8)

(ii) Obtain the general solution of $(D^2 - 2DD' + D'^2)z = \sin x$. (8)

12. (a) Obtain the Fourier series expansion of $f(x) = 2x - x^2$ in the interval $0 < x < 3$. (16)

Or

(b) The displacement y of a part of a mechanism is tabulated with corresponding angular movement x° of the crank. Express y as a Fourier series neglecting the harmonics above the third. (16)

x°:	0	30	60	90	120	150	180	210	240	270	300	330
y:	1.80	1.10	0.30	0.16	1.50	1.30	2.16	1.25	1.30	1.52	1.76	2.00

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13. (a) A tightly stretched string of length l is fastened at both ends. Initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement y at any distance x from one end at any time t. (16)

Or

(b) A bar with 100 cm long, with insulated sides, has its ends kept at 0°C and 100°C respectively until steady state conditions prevail. The two ends are then suddenly insulated and kept so. Then, find the temperature distribution function. (16)

14. (a) (i) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1-x^2, & for\ |x| \le 1 \\ 0, & for\ |x| > 1. \end{cases}$ Hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx.$$ (10)

(ii) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that $F\{f(x-a)\} = e^{isa} F(s)$. (6)

Or

(b) Find the Fourier transform of $f(x) = e^{-a^2x^2}$, $a > 0$. Hence deduce that $e^{\frac{-x^2}{2}}$ is self reciprocal in respect of Fourier transform. (16)

15. (a) Using Z transform, solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$. (16)

Or

(b) State and prove the convolution theorem in Z transforms and apply it to find $Z^{-1}\left\{\frac{z^2}{(z-2)(z-3)}\right\}$. (16)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 21279

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023

Third Semester

Civil Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Obtain the partial differential equations by eliminating arbitrary constants a and b from $(x+a)^2+(y-b)^2=z$.

2. Solve $(D^2-4DD'+3D'^2)z=0$.

3. State the condition for a function $f(x)$ to be expressed as a Fourier series.

4. If $f(x)=x^2+x$ is a expresses as a Fourier series in the interval (–2,2) to which value this series converges at $x=2$.

5. Classify the partial differential equation $u_{xx}-y^4u_{yy}=2y^3u_y$.

6. Write down the various solutions of one dimensional wave equation.

7. State the convolution theorem of Fourier Transform.

8. Prove that $F[f(x-a)]=e^{isa}F(s)$.

9. Find the Z transform of 1.

10. State initial and final value theorem of Z transforms.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the general solution of $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (8)

(ii) Solve the partial differential equation $[D^2 + 3DD' + 2D'^2]z = x + y$. (8)

Or

(b) (i) Obtain the singular solution of the partial differential equation. $(pq - p - q)(z - px - qy) = pq$. (8)

(ii) Solve the partial differential equation $[D^2 + 2DD' + D'^2]z = x^2y + e^{x-y}$. (8)

12. (a) (i) Expand the Fourier series for $f(x) = x(2\pi - x)$ in $(0, 2\pi)$ and hence deduce that the sum of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$. (10)

(ii) Expand the function $f(x) = x$, $0 < x < \pi$ in the Fourier sine series. (6)

Or

(b) (i) Obtain the function $f(x) = \sin x$, $0 < x < \pi$ in Fourier cosine series. (8)

(ii) Determine the first two harmonic of the Fourier series for the following values (8)

x :	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y :	1.98	1.30	1.05	1.30	–0.88	–0.25

13. (a) A string is tightly stretched and its ends are fastened at the two points $x = 0$ and $x = 2l$. The mid-point of the string is displaced transversely through a small distance b and the string is released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time.

Or

(b) The ends A and B of a rod 30 cm long have their temperature kept at 20°C. and 80°C respectively until steady state condition prevail. The temperature at each end is then suddenly reduced to 60°C and that of A is raised 40°C. Find the temperature distribution in the rod after time t.

14. (a) (i) Find the Fourier sine and cosine transform of e^{-ax} and hence deduce the inversion formula. (8)

(ii) Find the Fourier integral of the function $f(t)=\begin{cases} e^{at}, & t<0 \\ e^{-at}, & t>0 \end{cases}$. (8)

Or

(b) Obtain the Fourier transform of $f(x)=\begin{cases} 1-|x|, & if\ |x|<1 \\ 0, & if\ |x|>1 \end{cases}$ Hence deduce that $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$. Using Parseval's identity find the value of $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^4 dt$.

15. (a) (i) Find the $Z-$transforms of $\cos\left(\frac{n\pi}{2}\right)$ and $\frac{2n+3}{(n+1)(n+2)}$. (8)

(ii) Obtain the inverse Z-transforms of $\frac{z^2+2z}{z^2+2z+5}$. (8)

Or

(b) (i) Find the inverse Z-transforms of $\frac{2z^2+3z}{(z+2)(z-4)}$. (8)

(ii) Solve the difference equation $y_{n+2}+4y_{n+1}+3y_n=3^n$ with $y_0=0$, $y_1=1$ by using Z transforms. (8)

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