CE3391 **FLUID MECHANICS AND MACHINERY**

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COURSE OBJECTIVES:

1. To introduce the students about properties of the fluids, behaviour of fluids under static conditions.
2. To impart basic knowledge of the dynamics of fluids and boundary layer concept.
3. To expose to the applications of the conservation laws to a) flow measurements b) flow through pipes (both laminar and turbulent) and c) forces on pipe bends.
4. To exposure to the significance of boundary layer theory and its thicknesses.
5. To expose the students to basic principles of working of hydraulic machineries and to design Pelton wheel, Francis and Kaplan turbine, centrifugal and reciprocating pumps.

UNIT I FLUID PROPERTIES AND FLOW CHARACTERISTICS 10+3

Properties of fluids – Fluid statics - Pressure Measurements - Buoyancy and floatation - Flow characteristics - Eulerian and Lagrangian approach - Concept of control volume and system - Reynold’s transportation theorem - Continuity equation, energy equation and momentum equation - Applications.

UNIT II FLOW THROUGH PIPES AND BOUNDARY LAYER 9+3

Reynold’s Experiment - Laminar flow through circular conduits - Darcy Weisbach equation - friction factor - Moody diagram - Major and minor losses - Hydraulic and energy gradient lines - Pipes in series and parallel - Boundary layer concepts - Types of boundary layer thickness.

UNIT III DIMENSIONAL ANALYSIS AND MODEL STUDIES 8+3

Fundamental dimensions - Dimensional homogeneity - Rayleigh’s method and Buckingham Pi theorem - Dimensionless parameters - Similitude and model studies - Distorted and undistorted models.

UNIT IV TURBINES 9+3

Impact of jets - Velocity triangles - Theory of rotodynamic machines - Classification of turbines - Working principles - Pelton wheel - Modern Francis turbine - Kaplan turbine - Work done - Efficiencies - Draft tube - Specific speed - Performance curves for turbines - Governing of turbines.

UNIT V PUMPS 9+3

Classification of pumps - Centrifugal pumps - Working principle - Heads and efficiencies– Velocity triangles - Work done by the impeller - Performance curves - Reciprocating pump working principle - Indicator diagram and it’s variations - Work saved by fitting air vessels - Rotary pumps.

TOTAL: 60 PERIODS

OUTCOMES:

On completion of the course, the student is expected to be able to

1. Understand the properties and behaviour in static conditions. Also, to understand the conservation laws applicable to fluids and its application through fluid kinematics and dynamics
2. Estimate losses in pipelines for both laminar and turbulent conditions and analysis of pipes connected in series and parallel. Also, to understand the concept of boundary layer and its thickness on the flat solid surface.
3. Formulate the relationship among the parameters involved in the given fluid phenomenon and to predict the performances of prototype by model studies
4. Explain the working principles of various turbines and design the various types of turbines.
5. Explain the working principles of centrifugal, reciprocating and rotary pumps and design the centrifugal and reciprocating pumps

TEXT BOOKS:

1. Modi P.N. and Seth, S.M. Hydraulics and Fluid Mechanics, Standard Book House, New Delhi, 22nd edition (2019)
2. Jain A. K. Fluid Mechanics including Hydraulic Machines, Khanna Publishers, New Delhi, 2014.
3. Kumar K. L., Engineering Fluid Mechanics, Eurasia Publishing House(p) Ltd. New Delhi, 2016.

IMPORTANT QUESTION & ANSWERS

UNIT –I

FLUID PROPERTIES AND FLOW CHARACTERISTICS

1. Explain the variation of viscosity with temperature

For liquids viscosity inversely vary with the temperature and for gases the viscosity varies directly with the temperature.

2. Define –Incompressible fluid.

The density of the fluid is not constant for the fluid, generally all the liquids are incompressible

3. State the assumption used in the derivation of the Bernoulli's equation.

1. The fluid is ideal
2. the flow is steady
3. the flow is incompressible
4. The flow is irrotational

4. What is cohesion and adhesion in fluids?

Cohesion is due to the force of attraction between the molecules of the same liquid. Adhesion is due to the force of attraction between the molecules of two different liquids or between the molecules of the liquid and molecules of the solid boundary surface

5. What is kinematics viscosity? State its units

It is defined as the ratio of dynamic viscosity (μ) to mass density (ρ). (m^2/sec)

6. State momentum of momentum equation?

It states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

7. Define density and specific weight.

a. Density is defined as mass per unit volume (kg/m^3)
b. Specific weight is defined as weight possessed per unit volume (N/m^3)

8. **What are the properties of real fluid?**
    1. It is compressible
    2. They are viscous in nature
    3. Shear force exists always in such fluid.
9. **Define Surface tension and Capillarity?**

    Surface tension is due to the force of cohesion between the liquid particles at the free surface.

    Capillary is a phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid.
10. **Differentiate absolute and gauge pressure?**

    Gauge pressure is measured by gauge which is above atmospheric pressure, absolute pressure which measured from absolute zero level.
11. **Determine the capillary rise of mercury in a 2 mm ID glass tube. Assume, σ = 0.5 N/m and β = 130°.**

    Specific weight of mercury, γ = 13600 X 9.81 N/m^3**,**

    Therefore, $h = (4\sigma \cos\beta)/\rho g D$

    $h = (4 \times 0.5 \times \cos 130)/13600 \times 9.8 \times 0.002$

    $h = -4.82 \times 10^{-3} = -4.82$ mm

PART-B

P

1. A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. Both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 r.p.m. determine the viscosity of the fluid. & Calculate the power? [May / June - 2013]

Given :

Diameter of cylinder = 15 cm = 0.15 m

Dia. of outer cylinder = 15.10 cm = 0.151 m

length of cylinders, L = 25 cm = 0.25 m.

Torque T = 12 Nm.

Speed N = 100 r.p.m.

To Find :

Viscosity μ = ? ; Power (P) = ?

Formula ;

$$\mu = \tau / \frac{du}{dy} \qquad \left[\because \tau = \mu . \frac{du}{dy} \right]$$

$$P = \frac{2\pi N T}{60}$$

Solution :

(i) Tangential Velocity of cylinder $u = \frac{\pi D N}{60}$

$$u = \frac{\pi \times 0.15 \times 100}{60} \Rightarrow 0.7854\ m/s.$$

(ii) Surface area of cylinder, $A = \pi D \times L$

$$A = \pi \times 0.15 \times 0.25 = 0.1178.$$

$$dy = \frac{0.151 - 0.15}{2} = 0.0005\,m.$$

$$\text{Shear stress}\,(\tau) = \mu \cdot \frac{du}{dy}$$

$$\tau = \frac{\mu \times 0.7854}{0.0005}$$

$$\underset{(F)}{\text{Shear force}} = \text{Shear } \underset{(\tau)}{\text{Stress}} \times \text{Area}\,(A)$$

$$\text{Torque}\,(T) = F \times D/2$$

$$12 = F \times \left(\frac{0.15}{2}\right)$$

$$F = \frac{12}{0.075} \Rightarrow 160\ N.$$

$$\boxed{F = 160\,N.}$$

$$\text{Shear Stress}\,(\tau) = \frac{F}{A}$$

$$= \frac{160}{0.1178} \Rightarrow 1358.23.$$

$$\boxed{\tau = 1358.23\ N/m^2}$$

$$\text{Viscosity}\ (\mu) = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

$$\tau = 1358.23\ N/m^2.$$

$$du = u - 0 \quad ; \quad u = 0.7854 \text{ m/s.}$$

$$dy = 0.0005 \text{ m.}$$

$$\mu = \frac{1358.23}{\left(\frac{0.7854}{0.0005}\right)} \Rightarrow 0.864 \text{ Ns/m}^2.$$

Viscosity $\mu = 0.864 \text{ Ns/m}^2.$ (or)

$$= 0.864 \times 10 \Rightarrow 8.64 \text{ poise.}$$

$$\text{Power (P)} = \frac{2\pi NT}{60}$$

$$= \frac{2 \times \pi \times 100 \times 12}{60.}$$

$$P = 125.66 \text{ W}$$

2. The Velocity distribution over a plate is given by the relation, $u = y\left(\frac{2}{3} - y\right)$; where y is the Vertical distance above the Plate in meters. Assuming a Viscosity of 0.9 Pa.s, find the shear stress at $y = 0$ and $y = 0.15$m. [NOV - Dec - 2012]

Given:

Velocity distribution $(u) = y\,(2/3 - y)$ (or)

$$\frac{2}{3}y - y^2.$$

Viscosity $(\mu) = 0.9$ Poise.

$$\left[1 \text{ poise} = \frac{1}{10} \frac{Ns}{m^2} \right.$$

$$0.9 \text{ poise} = \frac{0.9}{10}$$

$$\left. = 0.09\ Ns/m^2 \right]$$

To Find :

Shear Stress at a distance $y = 0$; $y = 0.15m$

Formula required:

$$\text{Shear Stress } (\tau)_{y=0;\ y=0.15m.} = \mu \cdot \frac{du}{dy}$$

Solution:

$$u = \frac{2}{3} y - y^2 \cdot \cdot \text{ [diff. w.r.t } y]$$

we get,

$$\frac{du}{dy} = \frac{2}{3} - 2y.$$

At $y = 0$;

$$\frac{du}{dy} = \frac{2}{3} - 2(0)$$

$$\frac{du}{dy} = \frac{2}{3}/s$$

(i) Shear Stress $(\tau)_{y=0} = \mu \cdot \left(\frac{du}{dy}\right)$

$$= 0.09 \times 2/3$$

$$(\tau)_{y=0} = 0.06 \ N/m^2 .$$

(ii) Shear Stress $(\tau)_{y=0.15} = \mu \cdot \left(\frac{du}{dy}\right)$

At $y = 0.15$,

$$\frac{du}{dy} = \frac{2}{3} - 2(0.15)$$

$$= 0.36 /s .$$

$$(\tau)_{y=0.15} = 0.09 \times 0.36$$

$$= 0.033 \ N/m^2 .$$

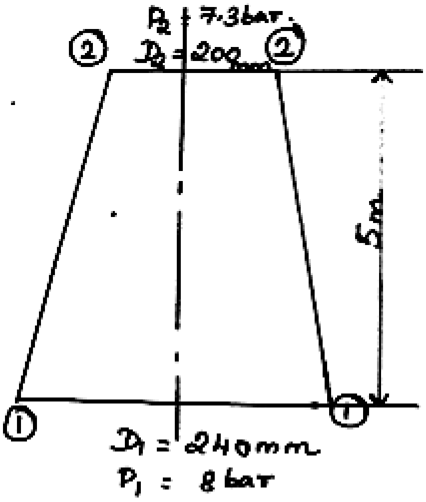
Result :

(i) Shear Stress at $y=0$ $(\tau)_{y=0}$ $= 0.06 \ N/m^2$.

(ii) Shear Stress at $y = 0.15m$ $(\tau)_{y=0.15}$ $\Big\} = 0.033 \ N/m^2$

3(a) Water flows at the rate of 200 litres per second upwards through a tapered vertical pipe. The diameter at the bottom is 240 mm and at the top 200 mm and the length is 5m. The Pressure at the bottom is 8 bar, and the Pressure at the topside is 7.3 bar. Determine the head loss through the pipe. Express it as a function of exit Velocity head. & (10) [NOV / DEC – 2014]
also the direction of flow.

Given:



$Q = 200$ lit/s $\Rightarrow 0.2\, m^3/s$.

$D_1 = 0.24$ m.

$D_2 = 0.2$ m.

$Z_2 = 5$m.

$P_1 = 8 \times 10^5$ N/mm²

$P_2 = 7.3 \times 10^5$ N/mm².

Find :

(i) Head Loss $(h_L) = ?$

(ii) Direction of flow : ?

Formula required :

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L.$$

Solution:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{8 \times 10^5}{\rho g} + \frac{V_1^2}{2g} + 0 = \frac{7.3 \times 10^5}{\rho \times g} + \frac{V_2^2}{2g} + 5 + h_L \rightarrow ①$$

$$Q = A_1 V_1 = A_2 V_2 .$$

$$V_1 = Q/A_1 = \frac{0.2}{\pi/4 \, (0.24)^2} \qquad \left[\begin{array}{l} A_1 = \pi/4 \, (d_1)^2 \\ A_2 = \pi/4 \, (d_2)^2 \end{array} \right]$$

$$= 4.42 \text{ m/s}.$$

$$V_2 = Q/A_2 = \frac{0.2}{\pi/4 \, (0.2)^2} = 6.36 \text{ m/s}.$$

Submit V_1 & V_2 Value on equation (1).

We get, $\left[\because \rho \text{ of water } 1000, \; g = 9.81 \right]$

$$\frac{8 \times 10^5}{1000 \times 9.81} + \frac{(4.42)^2}{2 \times 9.81} + 0 = \frac{7.3 \times 10^5}{1000 \times 9.81} + \frac{(6.36)^2}{2 \times 9.81} + 5 + h_l$$

$$81.5 + 0.995 = 74.4 + 2.06 + 5 + h_l$$

$$82.495 = 81.46 + h_L$$

$$h_L = 82.495 - 81.46$$

$$\boxed{h_L = 1.035 \text{ m}}$$

Express if a function of velocity head.

$$h_L = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$h_L = \frac{(6.36)^2}{2 \times 9.81} - \frac{(4.42)^2}{2 \times 9.81}$$

$$= 2.06 - 0.99$$

$$h_L = \frac{(6.36)^2}{2\times9.81} - \frac{(4.42)^2}{2\times9.81}$$

$$= 2.06 - 0.99$$

$$= 1.06\text{ m.}$$

$$\boxed{h_L = 1.06\text{ m}}$$

(ii) Direction of flow.

$$E_A = E_B + h_L \qquad \left[\because \begin{array}{l} E_A = 82.495 \\ E_B = 81.46 \end{array}\right]$$

$$h_L = E_A - E_B$$

As E_A is more than E_B and hence flow is taking Place from A to B.

Result :

(i) Loss of head $(h_L) = 1.035$ m.

(ii) Direction of flow = From A to B.

3(b) Determine the viscous drag torque & power absorbed on one surface of a collar bearing of 0.2 m ID & 0.3 m OD with an oil film thickness of 1mm & a viscosity of 30 centipoise if it rotates at 500. r.p.m. (6)

[NOV /DEC - 2014]

Given :

$D_i = 0.2\ m$

$D_0 = 0.3\ m$

$dy = 1\ mm.$

$\mu = 30\ c.p = 0.03\ Ns/m^2.$

$N = 500\ r.p.m.$

Find :

Drag Torque (T) : ?

Formula :

$$T = F \times D/2.$$

Solution :

(i) Velocity $u = \frac{\pi d_i N}{60}$

$$= \frac{\pi \times 0.2 \times 500}{60}$$

$$= 5.23\ m/s.$$

$du = u - 0$; $du = 5.23\ m/s.$

(ii) Shear Stress $\tau = \mu . \frac{du}{dy}$

$[\because \tau = F/A$

$F = \tau \times A]$

$$= 0.03 \times \frac{5.23}{0.001}$$

$$\boxed{\tau = 156.9\ N/mm^2}$$

(iii) Area of contact $(A) = 2\pi \times r \times l$. $\left[\because l = \frac{0.3-0.2}{2} = 0.05m\right]$

$$= 2\pi \times \left(\frac{0.2}{2}\right) \times 0.05$$

$$\boxed{\text{Area } (A) = 0.0314 m^2}$$

(iv) Force (F) = Shear Stress $(\tau) \times$ Area (A)

$$= 156.9 \times 0.0314$$

$$\boxed{F = 4.92 N.}$$

(v) Drag Torque $(T) = F \times \frac{D_1}{2}$.

$$= 4.92 \times \left(\frac{0.2}{2}\right)$$

Drag Torque (T) = 0.492 N-m.

Result :

Velocity (u) = 5.23 m/s

Shear Stress (τ) = 156.9 N/mm^2

Area of Contact (A) = 0.0314 m^2

Force (F) = 4.92 N.

Drag Torque (T) = 0.492 N-m.

4. A Pipe line of 175 mm diameter branches into two types which delivers the water at atmospheric pressure. The diameter of branch 1 which is at 35° counter clockwise to the pipe axis is 75mm & velocity at outlet is 15 m/s. The branch 2 is at 15° with the pipe center line in the clockwise direction. has a diameter of 100mm. The outlet velocity is 15 m/s. The pipes lie in a horizontal plane Determine the magnitude & direction of forces on the pipes. (16) [NOV/DEC - 2011].

Given:

Dia. of Main pipe $(d) = 175\,mm = 0.175\,m$.

Dia. of branch pipe 1 $(d_1) = 75\,mm = 0.75\,m$.

Velocity of branch pipe 1 $(V_1) = 15\,m/s$.

Dia. of branch pipe 2 $(d_2) = 100\,mm = 0.1\,m$.

Velocity of branch pipe 2 $(V_2) = 15\,m/s$.

Find:

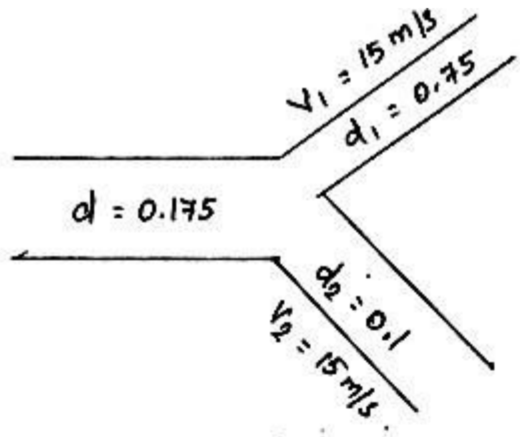
Determine magnitude & direction of forces.

Formula:

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

Solution:



By Continuity equation.

$$Q = Q_1 + Q_2$$

$$AV = A_1V_1 + A_2V_2. \qquad [\because Q = A \times V]$$

$$\frac{\pi}{4} d^2 \times V = \frac{\pi}{4} d_1^2 \times V_1 + \frac{\pi}{4} d_2^2 \times V_2. \qquad A = \pi/4\, d^2.$$

$$\frac{\pi}{4} \times 0.175^2 \times V = \frac{\pi}{4} \times 0.75^2 \times 15 + \frac{\pi}{4} \times 0.1^2 \times 15.$$

$$\boxed{V = 7.65 \text{ m/s}}$$

By resolving forces in x- direction,

$$F_x = F\cos\theta + F_1 \cos\theta_1 + F_2 \cos(360 - \theta_2) \rightarrow (1)$$

We Know that,

$$\underset{(F)}{\text{Force}} = \text{Mass} \times \text{acceleration}.$$

$$\text{Mass of water } (M) = \rho AV. \longrightarrow (2)$$

Substituting (2) in equ. (1)

$$F_x = \rho AV^2 \cos\theta + \rho A_1 V_1^2 \cos\theta_1 + \rho A_2 V_2^2 \cos(360 - \theta_2)$$

$$= 1000 \times \frac{\pi}{4} \times (0.175)^2 \times 7.65 \cos\theta.$$

$$F_x = 1000 \times \frac{\pi}{4} \times 0.075^2 \times 15 \cos 35^\circ$$

$$= 1000 \times \frac{\pi}{4} \times 0.1^2 \times 15 \cos(360 - 15^\circ).$$

$$\boxed{F_x = 352.08\ N.}$$

By resolving force in y-direction.

$$F_y = F \sin\theta + F_1 \sin\theta_1 + F_2 \sin(360 - \theta_2).$$

$$F_y = 1000 \times \frac{\pi}{4} \times 0.075^2 \times 15 \sin 35^\circ + 1000 \times \frac{\pi}{4} \times 0.1^2 \times$$

$$15 \sin(360 - 15)^\circ$$

$$\boxed{F_y = 7.52\ N}$$

$$F_R = \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{352.08^2 + 7.52^2}$$

$$\boxed{F_R = 352.16\ N.}$$

The direction of resultant force x. axis,

$$\tan\theta = \frac{F_y}{F_x}$$

$$= \frac{7.52}{352.08} \Rightarrow 0.0214$$

Result :

$$F_R = 352.16\ N.$$

$$\tan\theta = 0.0214.$$

5. A pipe 200m long slopes down at 1 in 100 and tapers from 600mm diameter at the lower end, and carries 100 lit/sec of oil having specific gravity 0.8. If the pressure gauge at the higher end reads 60 kN/m², determine the velocities at the two end. also the pressure at the lower end. Neglect all losses. (16)

[Apr/may - 2015]

Given:

$L = 200\,m.$

Slopes at $= 1/100$.

$D_1 = 600\,mm.$

$D_2 = 300\,mm.$

$Q = 100$ lit/sec $= 0.1\,m^3/s$.

$p_1 = 60 \times 10^3\,N/m^2$; $S = 0.8$.

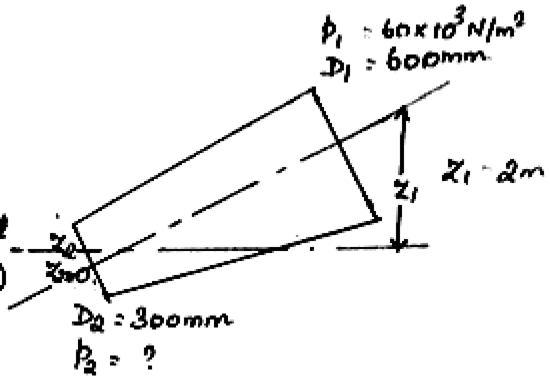
Find :

pressure at the lower end $(p_2) = ?$

formula required: Apply bernoulli's equation.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2.$$

Solution:



$z_2 = 0$ (b'coz it's located at Datum line)

$$z_1 = \frac{1}{100} \times 200 = 2m.$$

$$Q = A_1 V_1 = A_2 V_2.$$

$$Q = A_1 V_1$$

$$0.1 = \frac{\pi}{4}(d_1)^2 \times V_1 \;;\; \frac{\pi}{4} \times (0.6)^2 \times V_1$$

$$V_1 = \frac{0.1}{\frac{\pi}{4}(0.6)^2} \Rightarrow \frac{0.1}{0.2827} \Rightarrow 0.353\ m/s$$

$$\boxed{V_1 = 0.353\ m/s}$$

$$Q = A_2 V_2.$$

$$0.1 = \frac{\pi}{4}(d_2)^2 \times V_2 \;;\; \frac{\pi}{4} \times (0.3)^2 \times V_2.$$

$$V_2 = \frac{0.1}{\frac{\pi}{4}(0.3)^2} \Rightarrow \frac{0.1}{0.0706} \Rightarrow 1.4164\ m/s.$$

$$V_2 = 1.4164\ m/s.$$

Apply bernoulli's equation,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2.$$

$$\frac{60 \times 10^3}{1000 \times 9.81} + \frac{(0.353)^2}{2 \times 9.81} + 2 = \frac{p_2}{\rho g} + \frac{(1.416)^2}{2 \times 9.81} + 0$$

$$6.116 + \frac{0.1246}{19.62} + 2 = \frac{p_2}{\rho g} + \frac{2.005}{19.62} + 0$$

$$6.116 + 6.35 \times 10^{-3} + 2 = \frac{p_2}{\rho g} + 0.102 + 0.$$

$$8.12 = \frac{p_2}{\rho g} + 0.102 + 0$$

$$\frac{p_2}{\rho g} = 8.12 - 0.102$$

$$p_2 = 8.018 \times \rho \times g.$$

$$= 8.018 \times 1000 \times 9.81$$

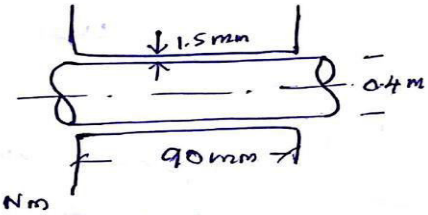
$$= 78656.58 \ N/m^2 \text{ (or)}$$

$$= 78.65 \ KN/m^2$$

Result :

Pressure at the lower end (p_2) = $78.65 \ KN/m^2$.

2. The dynamic viscosity of an oil used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.



$$\text{Power} = \frac{2\pi N T}{60} = 716.48\,W$$

$$T = \text{Force} \times \frac{D}{2} = 36.01\,Nm$$

$$\text{Force} = \text{shear stress} \times \text{Area}$$

$$= 1592 \times \pi D L$$

April/May 2017

$$\text{shear stress}, \tau = \mu \frac{du}{dy} = 1592\,N/m^2$$

$$du = u - 0 = 3.98\,m/s.$$

$$dy = t = 1.5 \times 10^{-3}\,m$$

$$\text{Tangential Velocity}, u = \frac{\pi D N}{60} = 3.98\,m/s$$

PART C

1. Explain Reynold's experiment. (Nov/Dec 2016)

In 1880's, Professor Osborne Reynolds carried out numerous experiment on fluid flow. We will now discuss the laboratory set up of his experiment. The experimental set used by Prof. Osborne Reynold is shown in Fig 1. As you can see from the figure, Reynolds injected dye jet in a glass tube which is submerged in the large water tank. Please see that the other end of the glass tube is out of water tank and is fitted with a valve. He made use of the valve to regulate the flow of water. The observations made by Reynolds from his experiment are given shown through Figures 5 to 7.

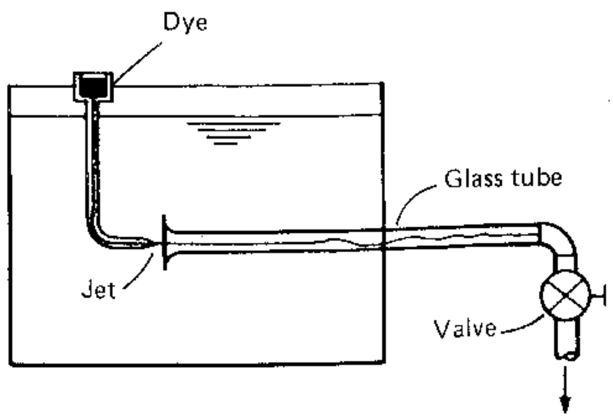


Fig. 1 Experimental Set up for Reynold's Experiment

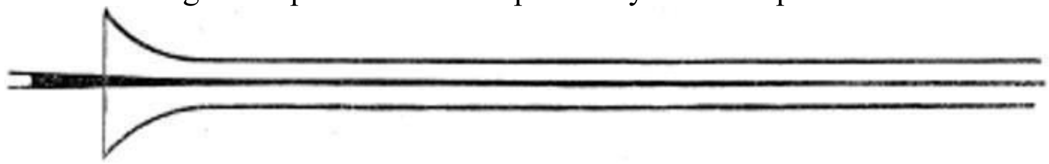


Fig. 2 Sketch showing the flow to be simple and ordered at low velocity

Fig. 3 The flow of dye forming wavy pattern at medium velocity

Fig. 4 The flow of dye is complex at higher velocity

APPROACH TOWARDS REYNOLDS' NUMBER

Throughout the experiment, Reynolds thought that the flow must be governed by a dimensionless quantity. What he observed was that Inertial force/Viscous force is unit less (dimensionless). Let us see the mathematical expression of inertial force and viscous force.

Inertial force is the force due to motion i.e. which may be also called as kinetic force.

Kinetic energy = $\frac{1}{2} mv^2$

Inertia force = $\rho v^2/2$

Viscous force = μ (du/dy)

Reynold's Number = Inertia force/ Viscous force

= $\rho v^2 dy/\mu\ du$

Now, for a finite length we can write dy = l, and du = v

Reynold's Number = Inertia force/ Viscous force

Reynold's number = $\rho vl/\mu$

UNIT –II

FLOW THROUGH PIPES AND BOUNDARY LAYER

1. **Difference between hydraulic Gradient line and Energy Gradient line.**

   Hydraulic gradient line :-

   Hydraulic gradient line is defined as the line which gives the sum of pressure head and datum head of a flowing fluid in a pipe with respect the reference line

   Total energy line :-

   Total energy line is defined as the line which gives the sum of pressure head , datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line

2. **Mention the general characteristics of laminar flow.**
   1. There is a shear stress between fluid layers
   2. 'No slip' at the boundary
   3. The flow is rotational
   4. There is a continuous dissipation of energy due to viscous shear

3. **Define boundary layer thickness**

   It is defined as the distance from the solid boundary in the direction perpendicular to the direction of flow where the velocity of fluid is approximately equal to 0.99 times the free stream velocity

4. **What is Hagen poiseuille's formula ?**

   $P1-P2 / pg = h\ f = 32\ \mu UL / \_gD^2$

   The expression is known as Hagen poiseuille formula .

   Where $P1-P2 / \_g$ = Loss of pressure head U = Average velocity

   μ = Coefficient of viscosity D = Diameter of pipe

   L = Length of pipe

5. **What is the expression for head loss due to friction in Darcy formula?**

   $hf = 4fLV^2 / 2gD$

Where f = Coefficient of friction in pipe L = Length of the pipe

D = Diameter of pipe V = velocity of the fluid

6. List the minor energy losses in pipes?

This is due to

i. Sudden expansion in pipe

ii. Sudden contraction in pipe.

iii. Bend in pipe.

iv. Due to obstruction in pipe

7. What are the factors influencing the frictional loss in pipe flow?

Frictional resistance for the turbulent flow is

1. Proportional to vn where v varies from 1.5 to 2.0.
2. Proportional to the density of fluid.
3. Proportional to the area of surface in contact.
4. Independent of pressure. Depend on the nature of the surface in contact.

8. What are the basic educations to solve the problems in flow through branched pipes?

i. Continuity equation.

ii. Bernoulli's formula.

iii. Darcy weisbach equation.

9. What is Dupuit's equation?

$$(L_1/d_1^5)+(L_2/d_2^5)+(L_3/d_3^5) = (L / d^5)$$

Where

L1, d1 = Length and diameter of the pipe 1

L2, d2 = Length and diameter of the pipe 2

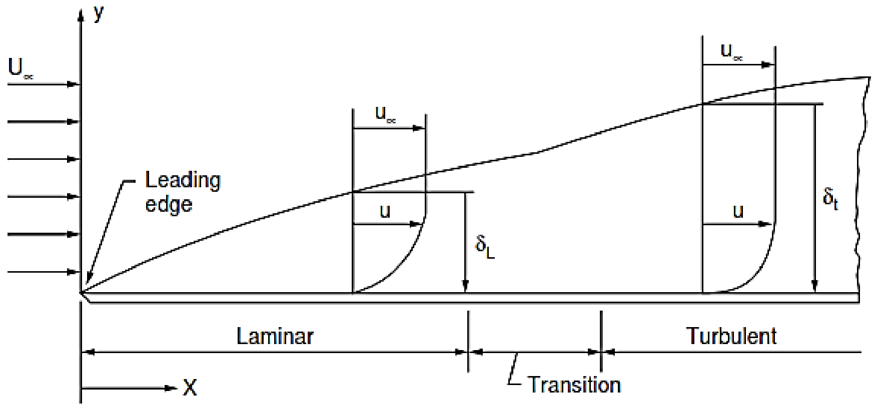
L3, d3 = Length and diameter of the pipe 3

10. Define Moody diagram

It is a graph in non-dimensional form that relates the Darcy friction factor, Reynolds number and relative roughness for fully developed flow in a circular pipe.

11. Define boundary layer. (April/May 2017)

When fluids flow over surfaces, the molecules near the surface are brought to rest due to the viscosity of the fluid. The adjacent layers also slow down, but to a lower and lower extent. This slowing down is found limited to a thin layer near the surface. The fluid beyond this layer is not



Affected by the presence of the surface. The fluid layer near the surface in which there is a general slowing down is defined as boundary layer.

12.What are equivalent pipes? Mention the equation used for it. (April/May 2017)

Equivalent pipes are defined as the pipes of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe.

The equation used to represent equivalent pipe is called Dupit's equation which is given as,

$$(L_1/d_1^5) + (L_2/d_2^5) + (L_3/d_3^5) = (L / d^5)$$

Where •

L1, d1 = Length and diameter of the pipe 1

L2, d2 = Length and diameter of the pipe 2

L3, d3 = Length and diameter of the pipe 3

PART-B

1. A laminar flow is taking place in a pipe at dia 20cm. The maximum velocity 1.5 m/s. Find mean velocity and radius at which this occurs. Also calculate velocity at 4 cm from wall of pipe. (16)

[Nov/Dec-2013]

Given:

$$D = 20\,cm = 0.20\,m.$$

$$U_{max} = 1.5\,m/s.$$

Find: (i) mean velocity, $\bar{u}$.

(ii) Radius at which $\bar{u}$ occurs.

(iii) velocity at 4cm from the wall.

Solution:

(i) Ratio of $\frac{U_{max}}{\bar{u}} = 2.0$ [Taken from the Derivation]

$$\frac{1.5}{\bar{u}} = 2$$

$$\bar{u} = \frac{1.5}{2} = 0.75\,m/s$$

$$\boxed{\bar{u} = 0.75\,m/s}$$

(ii) Radius at which $\bar{u}$ occurs.

The velocity u, at any radius 'r' is given by

$$u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)[R^2 - r^2] \text{ (or)}$$

$$-\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) R^2 \left[1 - r^2/R^2\right]$$

But from equation U_{max} is given by

$$U_{max} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \cdot R^2.$$

$$\left[\because u = U_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\right]$$

Now, the radius at which $u = \bar{u} = 0.75$ m/s.

$$0.75 = 1.5\left[1-\left(\frac{r}{(D/2)}\right)^2\right]$$

$$= 1.5\left[1-\left(\frac{r}{0.2/2}\right)^2\right]$$

$$= 1.5\left[1-\left(r/0.1\right)^2\right]$$

$$\frac{0.75}{1.5} = 1-\left(r/0.1\right)^2$$

$$\frac{r}{0.1} = 1-\frac{0.75}{0.1.5} = 1-\frac{1}{2} = 1/2$$

$$r = 0.1 \times \sqrt{0.5}$$

$$= 0.1 \times 0.707 = 0.0707 \text{ m}.$$

$$\boxed{r = 70.7 \text{ mm}.}$$

(iii) Velocity at 4cm from the wall,

$$r = R - 4.0$$

$$= 10-4 \Rightarrow 6.\text{cm (or) } 0.06\text{m}.$$

The velocity at a radius = 0.06 m (or)
4 cm from pipe wall is given by

$$= U_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$$

$$= 1.5\left[1-\left(\frac{0.06}{0.1}\right)^2\right]$$

$$= 1.5\left[1.0 - 0.36\right]$$

$$= 1.5 \times 0.64 = 0.96 \text{ m/s.}$$

$$\boxed{u = 0.96 \text{ m/s.}}$$

Result:

Mean velocity $\bar{u}$: 0.75 m/s.

radius at which $\bar{u}$ occurs (r) = 70.7 mm.

velocity at 4 cm from the wall (u) = 0.96 m/s.

•

2. An oil of specific gravity 0.80 & Kinematic viscosity $15 \times 10^{-6}\ m^2/s$. flows in a smooth pipe of 12 cm diameter at a rate of 150 lit/min. Determine whether the flow is laminar or turbulent. Also calculate the velocity at the center line & velocity at a radius of 4 cm. What a head loss for a length of 10m? What will be the entry length? Also determine the wall shear (16)
[Nov/Dec - 2014]

Given:

$$S = 0.80$$

$$\nu = 15 \times 10^{-6}\ m^2/s$$

$$d = 0.12\ m$$

$$Q = 150\ l/min = \frac{15 \times 10^{-3}}{60} = 0.0025\ m^3/s$$

$$r = 4\ cm = 0.04\ m$$

$$(x_2 - x_1)\quad dx = 10\ m$$

Find:

head loss $h_f = ?$

Entry length $= ?$

Wall shear $\tau_0 = ?$

Solution:

(i) $Re = \frac{VD}{\nu}$

$$Q = A \times V$$

$$V = Q/A$$

$$V = \frac{0.0025}{\pi/4\,(0.12)^2}$$

$$\boxed{V = 0.221\ m/s}$$

$$Re = \frac{0.221 \times 0.12}{15 \times 10^{-6}} \Rightarrow 1768.3$$

$Re < 2000$; $\therefore$ The flow is laminar.

(ii) $$U_{max} = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \cdot R^2.$$

$$p_1 - p_2 = \frac{32\mu \bar{u} L}{D^2} \qquad \frac{\partial p}{\partial x} = \frac{p_2 - p_1}{x_2 - x_1}.$$

$$\boxed{\bar{u} = V = 0.221\ m/s}$$

$$p_1 - p_2 = \frac{32 \times 0.012 \times 0.221 \times 10}{(0.12)^2}$$

[$\because V = m/\rho$

$m = V \times \rho$

$m = 15 \times 10^{-6} \times \rho$

$\rho = 1000 \times 0.8$

$= 800\ kg/m^3$]

$$p_1 - p_2 = 58.946\ N/m^2.$$

$$U_{max} = \frac{1}{4 \times 0.012} \times \frac{58.94}{10} \times 0.06^2$$

$$U_{max} = 0.441\ m/s$$

(iii) Velocity at 4 cm from center.

$$r = 0.04 \text{ m}.$$

$$= U_{max}\left(1 - \left(r/R\right)^2\right).$$

$$= 0.441\left(1 - \left(\frac{0.04}{0.06}\right)^2\right)$$

$$= 0.245 \text{ m/s}.$$

(iii) wall shear.

$$\tau_0 = -\left(\frac{\partial p}{\partial x}\right) \times \left(R/2\right)$$

$$= \frac{p_1 - p_2}{x} \times \left(R/2\right)$$

$$= \frac{58.94}{10} \times \frac{0.06}{2}$$

$$\tau_0 = 0.1767 \text{ N/m}^2.$$

V) Head loss for length 10m.

$$h_f = \frac{32 \mu \bar{u} L}{\rho g D^2}$$

$$= \frac{32 \times 0.012 \times 0.221 \times 10}{800 \times 9.81 \times (0.12)^2}$$

$$\boxed{h_f = 0.0075 \text{ m}}$$

Result :

(i) $h_f = 0.0075$ m.

(ii) wall shear $(\tau_0) = 0.1767$ N/m^2.

(iii) The velocity at 4 cm from center (u) $\Big\} = 0.245$ m/s.

3. Oil flows through a pipe 150mm in diameter and 650mm in length with a velocity of 0.5 m/s. If the Kinematic viscosity of oil is 18.7×10^{-4} m^2/s. Find the power lost in over coming friction. Take sp.g. of oil as 0.9. (16) [Apr/may - 2015]

Given :

$d = 150mm = 0.15$

$L = 650mm = 0.65$

$V = 0.5 m/s$

$\nu = 18.7 \times 10^{-4} m^2/s$

$S = 0.9$

$\left[\because \rho = 0.9 \times 1000 = 900 kg/m^3\right]$

Find :

Power lost (P)

Formula :

$$P = \frac{\rho Q g h_f}{1000} \text{ kW.}$$

Solution:

$$Re = \frac{VD}{\nu}$$

$$= \frac{0.5 \times 0.15}{18.7 \times 10^{-4}} \Rightarrow \frac{0.075}{18.7 \times 10^{-4}}$$

$$Re = 40.106 < 2000.$$ [Re Value is less than 2000]

The flow is laminar.

$$h_f = \frac{4 f L V^2}{2g \times d}.$$

[∵ if the flow is laminar Calculate $f = 16/Re$]

$$f = \frac{16}{Re}$$

$$= \frac{16}{40.106}$$

$$f = 0.3$$

$$h_f = \frac{4 \times 0.3 \times 650 \times (0.5)^2}{0.15 \times 2 \times 9.81}$$

$$h_f = \frac{195}{2.943} \Rightarrow 66.25$$

$$\boxed{h_f = 66.25 \text{ m}}$$

$$\text{Power lost } (P) = \frac{\rho g Q h_f}{1000} \text{ kW}$$

$$= \frac{9.81 \times 1900 \times 0.0088 \times 66.25}{1000}$$

$$\boxed{P = 5.147 \text{ kW}}$$

Result : (i) Head Loss $(h_f) = 66.25\,m$
(ii) Power lost $(P) = 5.147\,kW$.

4. Two pipes of dia 40cm & 20 Cm are each 300m long. when pipes connected in series & $0.10\,m^3/s$. Find loss of head & loss of head in S/m to pass the same total discharge when pipes connected in parallel Take $f = 0.0075$ for each pipe. (16)
[Nov/Dec - 2010]

Given:

$$D_1 = 40\,cm = 0.4\,m.$$
$$D_2 = 20\,cm = 0.2\,m.$$
$$L_1 = L_2 = 300\,m.$$
$$Q = 0.1\,m^3/s.$$
$$f = 0.0075.$$

Find :
(i) head loss for series & parallel.

Solution:

For series connection,

$$Q = A_1 V_1 = A_2 V_2$$

$$Q = A_1 V_1$$
$$0.1 = \frac{\pi}{4}(0.4)^2 \times V_1$$
$$\boxed{V_1 = 0.79\,m/s}$$

$$Q_2 = A_2 V_2$$
$$0.1 = \frac{\pi}{4}(0.2)^2 \times V_2$$
$$\boxed{V_2 = 3.18\,m/s}$$

Neglecting the minor losses.

$$H = \frac{4fL_1V_1^2}{2gd_1} + \frac{4f.L_2V_2^2}{2gd_2}.$$

$$= \frac{4 \times 0.0075 \times 300 \times (0.79)^2}{2 \times 9.81 \times 0.4} + \frac{4 \times 0.0075 \times 800 \times (3.18)^2}{2 \times 9.81 \times 0.2}$$

$$H = 0.715 + 23.19$$

$$\boxed{H = 23.4\,m}$$

For parallel connection,

$$h_f = \frac{4fL_1V_1^2}{2g \times d_1} = \frac{4fL_2V_2^2}{2g \times d_2}$$

$$\frac{V_1^2}{D_1} = \frac{V_2^2}{D_2}$$

$$\frac{V_1^2}{0.4} = \frac{V_2^2}{0.2}$$

$$V_1 = 1.41.V_2$$

$$Q = A_1V_1$$

$$0.1 = \pi/4\,(0.4)^2 \times V_1$$

$$= \pi/4\,(0.4) \times V_1$$

$$V_1 = 0.79\,m/s$$

$$Q = A_1 V_1 = A_2 V_2.$$

$$Q = A_2 V_2$$

$$\boxed{V_2 = 0.56 \text{ m/s}}$$

$$h_f = \frac{4 f L_1 V_1^2}{2g \times d_1}$$

$$= \frac{4 \times 0.0075 \times 300 \times (0.79)^2}{2 \times 9.81 \times 0.4}$$

$$\boxed{h_f = 0.71 \text{ m}}$$

Result;

Head Loss for series pipe is 23.9 m

Head Loss for parallel pipe is 0.71 m.

5. A pipe line of 0.6m diameter is 1.5 Km long. To Increase the discharge, another line of the same diameter is Introduced Parallel to the first in the second half of the length. Neglecting minor Losses. find the Increase in discharge if 4f = 0.04. The head at Inlet is 300mm. (16)

[Apr/may - 2015)

Given.

Dia. of pipe line (D) = 0.6m

Length of pipe line (L) = 1.5 Km

= 1.5x1000 = 1500m

4f = 0.04 (OR)

$f = 0.01$

Head at Inlet $h = 300mm = 0.3m$

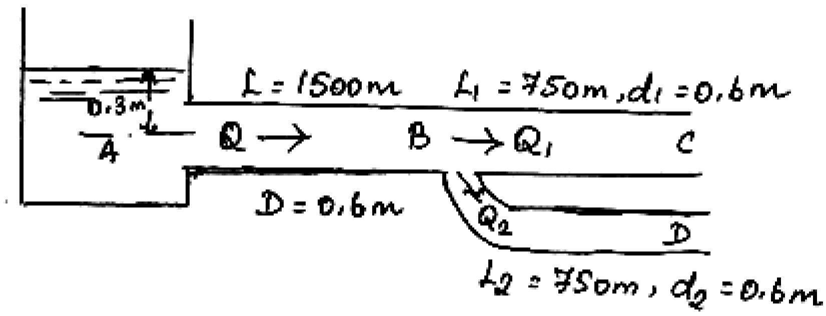
Head at outlet $=$ atmospheric head $= 0$

$\therefore$ Head loss $(h_f) = 0.3m$

Length of another parallel pipe $L_1 = \frac{1500}{2}$

$= 750m.$

Dia. of another parallel pipe. $d_1 = 0.6m.$



I st case.

Discharge for a single pipe of length 1500m & dia $= 0.6m$

This head lost due to friction in single pipe is $h_f = \frac{4fLV^2}{d \times 2g}.$

Where $V^* =$ Velocity of flow for single pipe.

$$0.3 = \frac{4 \times 0.01 \times 1500 \times V^{*2}}{0.6 \times 2g}$$

$$V^* = \sqrt{\frac{0.3 \times 0.6 \times 2 \times 9.81}{4 \times 0.01 \times 1500}} \Rightarrow 0.2426 \, m/s.$$

Discharge $Q^* =$ Area $\times V^*$

$= 0.2426 \times \pi/4 \, (0.6)^2$

$= 0.0685 \, m^3/s.$

2^{nd} case.

When an additional pipe of length 750m & diameter 0.6 m is connected in parallel with the last half length of the pipe.

Let, $Q_1 \rightarrow$ discharge in 1^{st} parallel pipe.

$Q_2 \rightarrow$ discharge in 2^{nd} parallel pipe

$$Q = Q_1 + Q_2.$$

where, $Q \rightarrow$ discharge in mainpipe when pipes are parallel.

But as the length & diameters of each parallel pipe is same.

$$\because \quad Q_1 = Q_2 = Q/2$$

Consider the flow through pipe ABC or ABD

$$\text{Head loss through } ABC = \underset{\text{through } AB}{\text{Head lost}} + \underset{\text{through } BC}{\text{Head lost}} \rightarrow (1)$$

ut head lost due to friction through ABC = 0.3 m given.

$$\text{Head loss due to friction through } AB = \frac{4 \times f \times 750 \times V^2}{0.6 \times 2 \times 9.81},$$

where V = Velocity of flow through AB.

$$= \frac{Q}{Area} = \frac{Q}{\pi/4\,(0.6)^2} = \frac{4Q}{\pi \times 0.36}$$

$\therefore$ Head loss due to friction through AB

$$= \frac{4 \times 0.01 \times 750}{0.6 \times 2 \times 9.81} \times \left(\frac{4Q}{\pi \times 0.36}\right)^2$$

$$= 31.87\,Q^2$$

Head loss due to friction through BC

$$= \frac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g}$$

$$= \frac{4 \times 0.01 \times 750}{0.6 \times 2 \times 9.81} \times \left[\frac{Q}{2 \times \pi/4 (0.6)^2} \right]$$

$$\left[\because V_1 = \frac{\text{Discharge}(Q)}{\text{Area}(A)} = \frac{Q}{\pi/4 (0.6)^2} \right]$$

$$= \frac{4 \times 0.01 \times 750}{0.6 \times 2 \times 9.81} \times \frac{16}{4 \times \pi^2 \times 0.36^2} Q^2$$

$$= 7.969\, Q^2$$

Substituting these values in equ (1) we get,

$$0.3 = 31.87\, Q^2 + 7.969\, Q^2$$

$$= 39.839\, Q^2$$

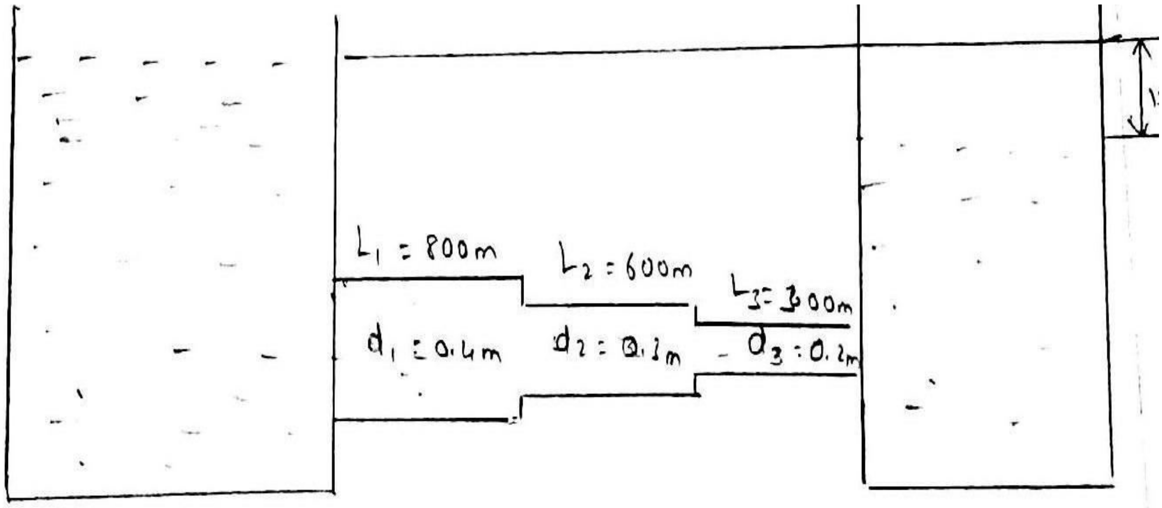
$$Q = \sqrt{\frac{0.3}{39.839}} = 0.0867\ m^3/s.$$

$\therefore$ Increase in discharge $= Q - Q^*$

$$= 0.0867 - 0.0685$$

$$= 0.0182\ m^3/s.$$

8. **Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400 m, 200 m, and 300 m respectively. They are connected in series to make a compound pipe. The ends of the compound pipe are connected with two tanks whose difference of water levels is 16 m. if the coefficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first the minor losses and then including them.** ***(Nov/Dec 2016)***



Given:

Length of pipe ①, $L_1 = 800\ m$

Length of pipe ②, $L_2 = 600\ m$

Length of pipe ③, $L_3 = 300\ m$

Diameter of pipe ①, $d_1 = 400\ mm = 0.4\ m$

Diameter of pipe ②, $d_2 = 300\ mm = 0.3\ m$

Diameter of pipe ③, $d_3 = 200\ mm = 0.2\ m$

Difference head at inlet and outlet, $H = 15\ m$

Soln:-

Given: Total head loss, $H = 15\,m$

$$\Rightarrow h_i + h_o + h_{c_2} + h_{c_3} + h_{f_1} + h_{f_2} + h_{f_3} = 15$$

$$\Rightarrow \frac{0.5V_1^2}{2g} + \frac{V_2^2}{2g} + \frac{0.5V_2^2}{2g} + \frac{0.5V_3^2}{2g} + \frac{4fL_1V_1^2}{2gd_1}$$

$$+ \frac{4fL_2V_2^2}{2gd_2} + \frac{4fL_3V_3^2}{2gd_3} = 15 \quad \text{--- (1)}$$

By continuity eqn:

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$$A_1V_1 = A_2V_2 \qquad\qquad A_3V_3 = A_2V_2$$

$$\frac{\pi}{4} \times 0.4^2\, V_1 = \frac{\pi}{4} \times 0.3^2\, V_2 \qquad\qquad \frac{\pi}{4} \times 0.2^2\, V_3 = \frac{\pi}{4} \times 0.3^2\, V_2$$

$$\boxed{V_1 = 0.562\, V_2} \qquad\qquad \boxed{V_3 = 2.25\, V_2}$$

Sub: V_1 and V_3 in (1)

$$\Rightarrow \frac{0.5(0.562\,V_2)^2}{2g} + \frac{(2.25V_2)^2}{2g} + \frac{0.5V_2^2}{2g} + \frac{0.5(2.25V_2)^2}{2g}$$

$$+ \frac{4 \times 0.005 \times 800 \times (0.562V_2)^2}{2g \times 0.4} + \frac{4 \times 0.005 \times 600 \times V_2^2}{2g \times 0.3}$$

$$+ \frac{4 \times 0.005 \times 300\,(2.25V_2^2)}{2g \ldots} = 15$$

$$\Rightarrow \frac{0.1579 V_2^2}{2g} + \frac{5.0625 V_2^2}{2g} + \frac{0.5 V_2^2}{2g} + \frac{2.53 V_2^2}{2g}$$

$$+ \frac{12.63 V_2^2}{2g} + \frac{40 V_2^2}{2g} + \frac{151.575 V_2^2}{2g} = 15$$

$$\Rightarrow \frac{212.75 V_2^2}{2g} = 15$$

$$\Rightarrow V_2^2 = \frac{15 \times 9.81 \times 2}{212.75}$$

$$\boxed{V_2 = 1.176} \ m/s$$

Discharge, $Q = A_2 \times V_2$

$$= \frac{\pi}{4} \times 0.2^2 \times 1.17$$

$$\boxed{Q = 0.083 \ m^3/sec}$$

W.K.T,

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{1700}{d^5} = \frac{800}{(0.4)^5} + \frac{600}{(0.3)^5} + \frac{300}{(0.2)^5}$$

$$\frac{1700}{d^5} = 78125 + 246913.5 + 937500$$

$$\frac{1700}{d^5} = 1262538.5$$

$$d^5 = 1.3468 \times 10^{-3}$$

$$d = 0.2665 \text{ m}$$

$$= 0.2665 \times 1000 \text{ mm}$$

$$\boxed{d = 266.5 \text{ mm}}$$

9. A fluid of viscosity 8 poise and specific gravity 1.2 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is $210 N/m^2$.

Find:

(i) The pressure gradient,

(ii) The average velocity and

(iii) Reynold's number of flow.

Solution:

Viscosity of fluid, $\mu = 8$ poise $= 0.8 \text{ Ns/m}^2$.

Specific gravity $= 1.2$

$\therefore$ Mass density, $\rho = 1.2 \times 1000 = 1200 \text{ kg/m}^3$.

Diameter of the pipe, $D = 100 \text{mm} = 0.1 \text{ m}$.

Maximum shear stress, $\tau_0 = 210 \text{ N/m}^2$

(i) The pressure gradient, $\frac{\partial p}{\partial x}$:

We know that, $\tau_0 = -\frac{\partial p}{\partial x} \cdot \frac{R}{2}$

$$210 = -\frac{\partial p}{\partial x} \cdot \frac{(0.1/2)}{2}$$

$$\Rightarrow \boxed{\frac{\partial p}{\partial x} = -8400 \text{ N/m}^2 \text{ per m.}}$$

(ii) The average velocity, $\bar{u}$:

We know that, $\bar{u} = \frac{1}{2} U_{max}$

$$= \frac{1}{2}\left[-\frac{1}{4\mu}\cdot\frac{\partial p}{\partial x}\cdot R^2\right]$$

$$= \frac{1}{2}\left[-\frac{1}{4\times 0.8}\times(-8400)\times(0.1/2)^2\right]$$

$$\boxed{\bar{u} = 3.28 \text{ m/s}}$$

PART- C

1. The velocity distribution in the boundary layer is given by,

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2,$$ δ being boundary layer thickness.

Nov/Dec 2016

Calculate the following:

(i) Displacement thickness.

(ii) Momentum thickness. &

(iii) Energy thickness.

Solution:

(i) Displacement thickness, δ^*:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left[1 - \left\{2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right\}\right] dy$$

$$= \int_0^\delta \left[1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right] dy$$

$$= \left[y - \frac{2}{2}\times\frac{y^2}{\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$\boxed{\delta^* = \delta/3}$$

(ii) <u>Momentum thickness</u>, θ:

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \left[\frac{2}{2} \times \frac{y^2}{\delta} - \frac{5}{3} \times \frac{y^3}{\delta^2} + \frac{4}{4} \times \frac{y^4}{\delta^3} - \frac{1}{5} \times \frac{y^5}{\delta^4}\right]_0^\delta$$

$$= \left[\delta - \frac{5}{3}\delta + \delta - \frac{1}{5}\delta\right]$$

$$\boxed{\theta = \frac{2}{15}\delta}$$

(iii) <u>Energy thickness, δ_e:</u>

$$\delta_e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy.$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right)\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{8y^3}{\delta^3} - \frac{2y^5}{\delta^5} + \frac{8y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5}\right] dy$$

$$= \left[\frac{2}{2} \times \frac{y^2}{\delta} - \frac{1}{3} \times \frac{y^3}{\delta^2} - \frac{8}{4} \times \frac{\cancel{8}y^4}{\delta^3} + \frac{12}{5} \times \frac{y^5}{\delta^4} - \frac{6}{6} \times \frac{y^6}{\delta^5} + \frac{1}{7} \times \frac{y^7}{\delta^6}\right]_0^\delta$$

$$= \left(\delta - \frac{\delta}{3} - 2\delta + \frac{12\delta}{5} - \delta + \frac{\delta}{7}\right)$$

$$\boxed{\delta_e = \frac{22\delta}{105}}$$

UNIT –III

DIMENSIONAL ANALYSIS

1. **Define dimensional homogeneity.**

   The dimensions of each term in an equation on both sides are equal. Thus if the dimensions of each term on both sides of an equation are the same the equation is known as dimensionally homogeneous equation

2. **Derive the expression for Reynold's number?**

   It is the ratio between inertia forces to the viscous force

   $Re = \rho v D / \mu$

3. **Define Mach number?**

   It is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force

4. **State the Buckingham's π theorem?**

   If there are n variables (dependent and independent) in a physical phenomenon and if these variables contain m fundamental dimensions, then these variables are arranged into (n-m) dimensionless terms called Pi terms

5. **Name the methods for determination of dimensionless groups.**

   i) Buckinghams pi theorem

   ii) Raleyritz method

5. State Froude's dimensionless number.

It is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force

- $F_e = \sqrt{\quad}$ i Fg.

7. **Define dynamic similarity.**

   Dynamic similarity is said to exist between the model and the prototype if the ratios of corresponding forces at the corresponding points in the model are the same.

8. What are the advantages of model and dimensional analysis?

1. The performance of the structure or the machine can be easily predicted.
2. With the dimensional analysis the relationship between the variables influencing a flow in terms of dimensionless parameter can be obtained.
3. Alternative design can be predicted and modification can be done on the model itself and therefore, economical and safe design may be adopted.

9. List the basic dimensional units in dimensional analysis.

1. Length(L)-meter
2. Mass(M)- kilogram
3. Time (T)- seconds

10. What are distorted models? State its merits and demerits.

A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratios for the linear dimensions are adopted.

Merits

1. The vertical dimensions of the model can be measured accurately
2. The cost of the model can be reduced
3. Turbulent flow in the model can be maintained.

Demerits

1. The results of the distorted model cannot be directly transferred to its prototype.

11. **Derive the scale ratio for velocity and pressure intensity using Froude model law. (Nov/Dec 2016)**

$$(F_e)_m = (F_e)_p \Rightarrow \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

Scale ratios based on froude number

(a) Scale ratio for time, $T_r = \frac{T_p}{T_m} = \sqrt{L_r}$

(b) Scale ratio for acceleration,

$$a_r = \frac{V_p}{V_m} \times \frac{T_m}{T_p} = \sqrt{L_r} \times \frac{1}{\sqrt{L_r}}$$

$$a_r = 1$$

Scale ratio for pressure intensity,

$$p = \frac{\text{Force}}{\text{Area}} = \frac{\rho L^2 V^2}{L^2}$$

$$p = \rho V^2$$

$$p_r = \frac{\rho_p}{\rho_m} \cdot \frac{V_p^2}{V_m^2}$$

for same fluid, $\rho_p = \rho_m$

$$p_r = \frac{V_p^2}{V_m^2} = \sqrt{L_r}^2 = L_r$$

$$\boxed{p_r = L_r}$$

PART-B

1. Using Buckingham's π- theorem, show that the velocity through a circular orifice is given by $V = \sqrt{2gH}\ \phi\left[D/H, \frac{\mu}{\rho V H}\right]$, where H is the head causing flow, D is the diameter of the orifice, μ is co-efficient of viscosity, ρ is the mass density and g is the acceleration due to gravity. (16)

[Apr/May - 2010].

Solution:

Given:

V is a function of H, D, μ, ρ and g

$$V = f(H, D, \mu, \rho, g) \text{ (or)} \rightarrow (i)$$

$$f_1(V, H, D, \mu, \rho, g) \rightarrow (ii)$$

Total no. of variable; $n = 6$.

dimensions of each variable,

$V = LT^{-1}$; $\mu = ML^{-1}T^{-1}$

$H = L$; $\rho = ML^{-3}$

$D = L$; $g = LT^{-2}$

No. of fundamental dimensions $m = 3$

$\therefore$ Number of π- terms $= n - m$

$= 6 - 3 = 3$

Equation (i) can be written as $f_1(\Pi_1, \Pi_2, \Pi_3) = 0$.

Each Π-term contains m+1 variables, where m = 3 and is also equal to repeating variables, Here V is a dependent variable and hence should not be selected as repeating variable. Choosing H, g, ρ as repeating variable, we get three Π-terms as,

$$\Pi_1 = H^{a_1} \cdot g^{b_1} \cdot \rho^{c_1} \cdot V \longrightarrow (1)$$

$$\Pi_2 = H^{a_2} \cdot g^{b_2} \cdot \rho^{c_2} \cdot D \longrightarrow (2)$$

$$\Pi_3 = H^{a_3} \cdot g^{b_3} \cdot \rho^{c_3} \cdot \mu \longrightarrow (3)$$

First Π-term :

$$\Pi_1 = H^{a_1} \cdot g^{b_1} \cdot \rho^{c_1} \cdot V.$$

Substituting dimensions on both sides,

$$M^0 L^0 T^0 = L^{a_1} \cdot (LT^{-2})^{b_1} \cdot (ML^{-3})^{c_1} \cdot LT^{-1}$$

Equating the power of M, L, T on both sides,

Power of M, $\boxed{0 = c.}$

Power of L, $0 = a_1 + b_1 - 3c_1 + 1$

$a_1 = -b_1 + 3c_1 - 1$

$= 1/2 + 0 - 1$; $\boxed{a_1 = -1/2}$

Power of T, $0 = -2b_1 - 1$

$\boxed{b_1 = -1/2}$

Third Π-term :

$$\Pi_3 = H^{a_3} \cdot g^{b_3} \cdot \rho^{c_3} \cdot \mu.$$

$$M^0 L^0 T^0 = L^{a_3} \cdot (LT^{-2})^{b_3} \cdot (ML^{-3})^{c_3} \cdot ML^{-1} T^{-1}$$

Equ.. the power of M, L, T on both,

Power of M : $0 = c_3 + 1$; $\boxed{c_3 = -1}$

Power of L : $0 = a_3 + b_3 - 3c_3 - 1$

$a_3 = -b_3 + 3c_3 + 1$; $\frac{1}{2} - 3 + 1 = -\frac{3}{2}$

$\boxed{a_3 = -3/2}$

Power of T $= 0 = -2b_3 - 1$; $\boxed{b_3 = -\frac{1}{2}}$

Sub. the abc values on π_3 term,

$$\pi_3 = H^{-3/2} \cdot g^{-1/2} \cdot \rho^{-1} \cdot \mu$$

$$\boxed{\pi_3 = \frac{\mu}{H^{3/2} \cdot \rho\sqrt{g}}} \quad \text{(or)} \quad \frac{\mu}{H\rho\sqrt{gH}} = \frac{\mu V}{H\rho V\sqrt{gH}}$$

[Multiply & $\div$ by V]

$$\boxed{\pi_3 = \frac{\mu}{H\rho V} \cdot \pi_1}$$

$\left[\because \frac{V}{\sqrt{gH}} = \pi_1\right]$

Substituting the values of π_1, π_2, π_3 in equation (ii)

$$f_1\left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \pi_1\frac{\mu}{H\rho V}\right) = 0 \quad \text{(or)}$$

$$\frac{V}{\sqrt{gH}} = \varphi\left[D/H, \pi_1\frac{\mu}{H\rho V}\right] \quad \text{(or)}$$

$$V = \sqrt{2gH}\ \varphi\left[D/H, \frac{\mu}{\rho V H}\right]$$

Multiplying by a constant doesnot change the character of π - terms.

2. The Power developed by hydraulic machine is found to depend on the head H, flowrate Q, density ρ, speed N, runner diameter D and acceleration due to gravity g. Obtain suitable dimensionless parameters to correlate experimental results. [16]

[NOV/DEC - 2014]
May/ June - 2012

Solution:

$$P = f(H, Q, \rho, N, D, g) \longrightarrow (i)$$

$$f_1(P, H, Q, \rho, N, D, g) = 0 \longrightarrow (ii)$$

Total no. of Variables $n = 7$.

No. of fundamental dimensions $m = 3$

$\therefore$ No. of π - terms $= n - m$

$= 7 - 3 \Rightarrow 4$.

$f_1 (\pi_1, \pi_2, \pi_3, \pi_4) = 0 \longrightarrow$ (iii)

$\pi_1 = H^{a_1} . N^{b_1} . \rho^{c_1} . P \longrightarrow$ (1)

$\pi_2 = H^{a_2} . N^{b_2} . \rho^{c_2} . Q \longrightarrow$ (2)

$\pi_3 = H^{a_3} . N^{b_3} . \rho^{c_3} . g \longrightarrow$ (3)

$\pi_4 = H^{a_4} . N^{b_4} . \rho^{c_4} . D \longrightarrow$ (4)

dimensions of each variables.

$H = L$; $N = T^{-1}$; $\rho = ML^{-3}$

$P = ML^2 T^{-3}$, $Q = L^3 T^{-1}$, $g = LT^{-2}$, $D = L$.

First π term:

$\pi_1 = H^{a_1} . N^{b_1} . \rho^{c_1} . P \longrightarrow$ (1)

applying dimensions on both sides

$M^0 L^0 T^0 = L^{a_1} . (T^{-1})^{b_1} . (ML^{-3})^{c_1} . ML^2 T^{-3}$.

Equating Power of M, L, T on both.

Power of M $= 0 = c_1 + 1$; $\boxed{c_1 = -1}$

Power of L $= 0 = a_1 - 3c_1 + 2$

$a_1 = 3c_1 - 2$

$= -3 - 2$

$\boxed{a_1 = -5}$

Power of T $= 0 = -b_1 - 3$

$$\boxed{b_1 = -3}$$

Substituting a_1, b_1, c_1 value in equation ①.

$$\pi_1 = H^{-5} \cdot N^{-3} \cdot \rho^{-1} \cdot P$$

$$\pi_1 = \frac{P}{H^5 N^3 \rho}$$

Second π-term:

$$\pi_2 = H^{a_2} \cdot N^{b_2} \cdot \rho^{c_2} \cdot Q \longrightarrow ②$$

applying dimension on both sides,

$$M^0 L^0 T^0 = (L)^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot L^3 T^{-1}.$$

Equating Power of M, L, T on both sides.

Power of M $= \boxed{c_2 = 0}$

Power of L $= a_2 - 3c_2 + 3 = 0$

$$a_2 = 3c_2 - 3$$

$$\boxed{a_2 = -3}$$

Power of T $= -b_2 - 1$

$$\boxed{b_2 = -1}$$

Substituting a_2, b_2, c_2 value in equation ②

$$\pi_2 = (H)^{-3} \cdot N^{-1} \cdot \rho^{0} \cdot Q$$

$$\boxed{\pi_2 = \frac{Q}{H^3 N}}$$

Third π-term:

$$\pi_3 = (H)^{a_3} \cdot (N)^{b_3} \cdot \rho^{c_3} \cdot g \longrightarrow ③$$

applying dimension on both sides,

$$M^0 L^0 T^0 = (L)^{a_3} \cdot (T^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot LT^{-2}$$

Power of M = $\boxed{c_3 = 0}$

Power of L = $a_3 - 3c_3 + 1$

$a_3 = 3c_3 - 1$; $\boxed{a_3 = -1}$

Power of T : $-b_3 - 2$; $\boxed{b_3 = -2}$

Substituting $a_3 . b_3 . c_3$ value in equ (3)

$$\pi_3 = H^{-1}, N^{-2}, \rho^{0} . g .$$

$$\boxed{\pi_3 = \frac{g}{N^2 H}}$$

Fourth π term :

$$\pi_4 = (H)^{a_4} . (N)^{b_4} . (\rho)^{c_4} . D \longrightarrow (4)$$

applying dimension on both sides,

$$M^0 L^0 T^0 = (L)^{a_4} . (T^{-1})^{b_4} . (ML^{-3})^{c_4} . L$$

equating the power of M, L, T on both.

Power of M = $\boxed{c_4 = 0}$

Power of L = $a_4 - 3c_4 + 1$

$a_4 = 0 - 1$

$\boxed{a_4 = -1}$

Power of T = $-b_4 = 0$. ; $\boxed{b_4 = 0}$

Substituting $a_4 . b_4 . c_4$ values on equ (4)

$$\pi_4 = H^{-1} . N^0 . \rho^0 . D$$

$$\boxed{\pi_4 = D/H}$$

Substitute π values in equation. (ii)

$$f(\pi_1, \pi_2, \pi_3, \pi_4) = 0 .$$

$$f\left[\frac{P}{H^5 . N^3 . \rho}, \frac{Q}{H^3 . N}, \frac{g}{N^2 H}, \frac{D}{H}\right] = 0.$$

$$\frac{P}{H^5 . N^3 . \rho} = \phi\left[\frac{Q}{H^3 . N}, \frac{g}{N^2 . H}, \frac{D}{H}\right]$$

$$P = H^5 N^3 \rho \;\phi\left[\frac{Q}{H^3 . N}, \frac{g}{N^2 . H}, \frac{D}{H}\right]$$

3. Derive on the basis of dimensional analysis suitable parameters to present the thrust developed by a propeller. Assume that the thrust P depends on the angular velocity ω, speed of advance V, diameter D, dynamic viscosity μ, mass density ρ, and elasticity of the fluid medium which can be denoted by the speed of sound in the medium 'c'. [16] [Nov/Dec – 2012]

Solution:

Thrust P is a function of $\omega, V, D, \mu, \rho, C$.

$$P = f(\omega, V, D, \mu, \rho, C) \longrightarrow (i)$$

$$f_1(P, \omega, V, D, \mu, \rho, C) = 0 \longrightarrow (ii)$$

$\therefore$ Total no. of variables $n = 7$.

dimensions of each variable,

$P = MLT^{-2}$; $\omega = T^{-1}$; $V = LT^{-1}$; $D = L$.

$\mu = ML^{-1}T^{-1}$; $\rho = ML^{-3}$; $C = LT^{-1}$

$\therefore$ No. of fundamental dimensions, $m = 3$.

Total. No. of π-terms $= n - m \Rightarrow 7 - 3$

$\Rightarrow 4$

Hence equation ① can be written as,

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0 \longrightarrow (iii)$$

$$\pi_1 = D^{a_1}.\ V^{b_1}.\ \rho^{c_1}.\ P$$

$$\pi_2 = D^{a_2}.\ V^{b_2}.\ \rho^{c_2}.\ \omega$$

$$\pi_3 = D^{a_3}.\ V^{b_3}.\ \rho^{c_3}.\ \mu.$$

$$\pi_4 = D^{a_4}.\ V^{b_4}.\ \rho^{c_4}.\ C$$

First π-term ;

$$\pi_1 = D^{a_1}.\ V^{b_1}.\ \rho^{c_1}.\ P \longrightarrow ①$$

applying dimentions on both sides,

Equating power of M, L, T on both sides.

Power of M $= 0 = c_1 + 1$; $\boxed{c_1 = -1}$

Power of L $= 0 = a_1 + b_1 - 3c_1 + 1$

$$a_1 = -b_1 + 3c_1 - 1$$

$$= 2 - 3 - 1 = -2. \quad \boxed{a_1 = -2}$$

Power of T $= 0 = -b_1 - 2$

$$\boxed{b_1 = -2}$$

Substituting the values of a_1, b_1 & c_1 in eqn ①

$$\pi_1 = D^{-2}.\ V^{-2}.\ \rho^{-1}.\ P$$

$$\boxed{\pi_1 = \frac{P}{D^2 V^2 . \rho}}$$

Second π-term : $\pi_2 = D^{a_2}.\ V^{b_2}.\ \rho^{c_2}.\ \omega$

applying dimention on both sides,

$$M^0 L^0 T^0 = L^{a_2}.\ (LT^{-1})^{b_2}.\ (ML^{-3})^{c_2}.\ T^{-1}$$

Equating the power of M, L, T on both,

Power of M $= 0 = c_2$; $\boxed{c_2 = 0}$

Power of L $= 0 = a_2 + b_2 - 3c_2$

$a_2 = -b_2 + 3c_2$.

$= 1 + 0 = 1$

$\boxed{a_2 = 1}$

Power of T $= 0 = -b_2 - 1$

$\boxed{b_2 = -1}$

Substituting the value of a_2, b_2, c_2 in π_2.

$\pi_2 = D^1 . V^{-1} . \rho^0 . \omega$

$$\boxed{\pi_2 = \frac{D\omega}{V}}$$

Third π-term:

$\pi_3 = D^{a_3} . V^{b_3} . \rho^{c_3} . \mu . \longrightarrow$ ③

applying dimension on both,

$M^0L^0T^0 = L^{a_3} . (LT^{-1})^{b_3} . (ML^{-3})^{c_3} . ML^{-1}T^{-1}$

Equating the power of M, L, & T on both,

Power of M $= 0 = c_3 + 1$; $\boxed{c_3 = -1}$

Power of L $= 0 = a_3 + b_3 - 3c_3 - 1$

$a_3 = -b_3 + 3c_3 + 1$

$= 1 - 3 + 1 = -1$

$\boxed{a_3 = -1}$

Power of T $= 0 = -b_3 - 1$

$\boxed{b_3 = -1}$

Substituting the values of a_3, b_3 & c_3 in π_3

$\pi_3 = D^{-1} . V^{-1} . \rho^{-1} . \mu$

$$\boxed{\pi_3 = \frac{\mu}{DV\rho}}$$

Fourth π - term :

$$\pi_4 = D^{a_4} \cdot V^{b_4} \cdot \rho^{c_4} \cdot c .$$

applying dimensions on both sides,

$$M^0 L^0 T^0 = L^{a_4} \cdot (LT^{-1})^{b_4} \cdot (ML^{-3})^{c_4} \cdot LT^{-1}$$

Equating the power of M, L, T on both sides.

Power of M $= 0 = c_4$; $\boxed{c_4 = 0}$

Power of L $= 0 = a_4 + b_4 - 3c_4 + 1$

$$a_4 = -b_4 + 3c_4 - 1$$

$$= 1 + 0 - 1 = 0$$

$$\boxed{a_4 = 0}$$

Power of T $= 0 = -b_4 - 1$

$$\boxed{b_4 = -1}$$

Substituting the values of a_4, b_4, c_4 in eqn (A)

$$\pi_4 = D^0 \cdot V^{-1} \cdot \rho^0 \cdot c = c/V$$

$$\boxed{\pi_4 = c/V}$$

Substituting the values of π_1, π_2, π_3 & π_4 in eqn (iii)

$$f_1 \left(\frac{P}{D^2 V^2 \cdot \rho} \cdot \frac{D\omega}{V}, \frac{\mu}{DV\rho} \cdot \frac{c}{V} \right) = 0 \quad (\text{or})$$

$$\frac{P}{D^2 V^2 \cdot \rho} = \phi \left[\frac{D\omega}{V}, \frac{\mu}{DV\rho}, \frac{c}{V} \right] \quad (\text{or})$$

$$P = D^2 V^2 \rho \ \phi \left[\frac{D\omega}{V}, \frac{\mu}{DV\rho}, \frac{c}{V} \right]$$

4. A Pipe of diameter 1.5m is required to transport an oil of sp. gr. 0.90 and viscosity 3×10^{-2} poise at the rate of 3000 l/s. Tests were conducted on a 15cm dia Pipe using water at 20°c. Find velocity & rate of flow in model. Viscosity of water at 20°c = 0.01 poise (16)

[NOV/DEC - 2012]

Given:

Dia of Prototype (D_p) = 1.5m.

Viscosity of prototype (μ_p) = 3×10^{-2} poise

Q_p = 3000 l/s ; 3 m^3/s

s_p = 0.9

$\therefore$ Density of Prototype (ρ_p) = $s_p \times 1000$

= 0.9×1000

= 900 kg/m^3.

Find:

Velocity & rate of flow in model.

V_m = ?

Q_m = ?

Formula required:

Using Reynold's model law.

$$\frac{\rho_m \cdot V_m \cdot D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$Q_m = A_m \times V_m$$

Solution:

For pipe flow, the dynamic similarity will be obtained if the Reynold's Number in the model & prototype are equal.

Hence using equation.,

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$\therefore \frac{V_m}{V_p} = \frac{\rho_p}{\rho_m} \cdot \frac{D_p}{D_m} \cdot \frac{\mu_m}{\mu_p}$$ [For pipe, linear Dimension is D]

$$= \frac{900}{1000} \times \frac{1.5}{0.15} \times \frac{1\times10^{-2}}{3\times10^{-2}}$$

$$= \frac{900}{1000} \times 10 \times \frac{1}{3} = 3.0$$

$$V_p = \frac{\text{Rate of flow in Prototype } (Q_p)}{\text{Area of Prototype } (A_p)} = \frac{3}{\pi/4 \, (D_p)^2}$$

$$V_p = \frac{3}{\pi/4 \, (1.5)^2} \Rightarrow \frac{3\times4}{\pi\times2.25} = 1.697 \text{ m/s}$$

$$V_m = 3\times V_p \Rightarrow 3\times1.697 = 5.091 \text{ m/s}$$

Rate of flow through model $(Q_m) = A_m \times V_m$

$$= \frac{\pi}{4}(D_m)^2 \times V_m$$

$$= \frac{\pi}{4}(0.15)^2 \times 5.091$$

$$= 0.0899 \text{ m}^3/\text{s}$$

$$= 0.0899 \times 1000 \text{ lit/s}$$

$$= 89.9 \text{ lit/s.}$$

Result :

(i) Velocity of model $(V_m) = 5.091$ m/s

(ii) Rate of flow through model $(Q_m) = 89.9$ lit/s.

5. The Efficiency η of a fan depends on the density ρ, the dynamic viscosity μ of the fluid. the angular velocity ω, diameter D of the rotor. and the discharge Q. Express η in terms of dimensionless parameters. Use Rayleigh's method. (16)

[Apr/May - 2015]

Solution:

$$\eta = K \cdot \rho^a \cdot \mu^b \cdot \omega^c \cdot D^d \cdot Q^e \longrightarrow (1)$$

Where K = Non dimensional constant.

dimensions of each variables.

$\rho = ML^{-3}$; $\mu = ML^{-1}T^{-1}$; $\omega = T^{-1}$;

$D = L$; $Q = L^3T^{-1}$

Substituting the dimensions on both sides. in equ (1)

$$M^0L^0T^0 = K\,(ML^{-3})^a \cdot (ML^{-1}T^{-1})^b \cdot (T^{-1})^c \cdot (L)^d \cdot (L^3T^{-1})$$

Power of M, $0 = a + b$

Power of L, $0 = -3a - b + d + 3e$

Power of T, $0 = -b - c - e$

Hence expressing a, c, & d in terms of b & e,

we get,

$a = -b$

$b = -(b+e)$

$d = 3a + b - 3e$

$= -3b + b - 3e$

$= -2b - 3e$.

Substituting a, b, d values in equation (1) we get,

$$\eta = K.\ \rho^{-b}.\ \mu^{b}.\ \omega^{-(b+e)}.\ D^{-2b-3e}.\ Q^{e}$$

$$= K.\ \rho^{-b}.\ \mu^{b}.\ \omega^{-b}.\ \omega^{-e}.\ D^{-2b}.\ D^{-3e}.\ Q^{e}$$

Result :

$$= K\left(\frac{\mu}{\rho\omega D^2}\right)^{b}.\left(\frac{Q}{\omega D^3}\right)^{e} = \varphi\left[\left(\frac{\mu}{\rho\omega D^2}\right).\left(\frac{Q}{\omega D^3}\right)\right]$$

6. Using Buckingham's π theorem, show that the velocity through a circular orifice is given by $V = \sqrt{2gH}\ \phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$, where H is the head causing flow, D is the diameter of the orifice, μ is the co-efficient of Viscosity, ρ is the mass density and g is the acceleration due to gravity.

April/May 2017

Solution : Given : V is a function of H, D, μ, ρ, g

$$V = f(H, D, \mu, \rho, g)$$

$$f_1(H, D, \mu, \rho, g, V) = 0 \quad \text{———} \quad (1)$$

(i) Total number of fundamental dimensions, m=3.

(ii) Total number of Variables, n=6

$\therefore$ Number of π-terms = n-m = 6-3 = 3.

Equation (1) can be written as,

$$f_1(\pi_1, \pi_2, \pi_3) = 0. \quad \text{———} \quad (2)$$

Each π term has m=3 repeating Variables and m+1 = 3+1 = 4 total Variables.

The repeating Variables are H, g, ρ

π terms can be written as,

$$\pi_1 = H^{a_1}\ g^{b_1}\ \rho^{c_1}\ V$$

$$\pi_2 = H^{a_2}\ g^{b_2}\ \rho^{c_2}\ D$$

$$\pi_3 = H^{a_3}\, g^{b_3}\, \rho^{c_3}\, \mu$$

Analysis of π terms:

First π term: $\pi_1 = H^{a_1}\, g^{b_1}\, \rho^{c_1}\, V$

substituting dimensions on both sides,

$$M^0 L^0 T^0 = L^{a_1} (LT^{-2})^{b_1} (ML^{-3})^{c_1} (LT^{-1})$$

Equating powers of M, L, T on both sides

Power of M, $0 = c$ $\therefore c_1 = 0$

Power of L, $0 = a_1 + b_1 - 3c_1 + 1$; $0 = a_1 + b_1 + 1$, $0 = a_1 - \frac{1}{2} + 1$

Power of T, $0 = -2b_1 - 1$, $2b_1 = -1$; $b_1 = -\frac{1}{2}$ $a_1 = -\frac{1}{2}$

substituting the value of a_1, b_1, c_1 in π_1

$$\pi_1 = H^{-1/2}\, g^{-1/2}\, \rho^0 . V$$

$$\pi_1 = \frac{V}{\sqrt{gH}}$$

Second π term:

substituting dimensions on both sides,

$$M^0 L^0 T^0 = L^{a_1} (LT^{-2})^{b_1} (ML^{-3})^{c_1}\, L$$

Equating powers of M, L, T on both sides

Power of M, $0 = c_1 \Rightarrow c_1 = 0$

Power of L, $0 = a_1 + b_1 - 3c_1 + 1 \Rightarrow 0 = a_1 + 1 \Rightarrow a_1 = -1$

Power of T, $0 = -2b_1 \Rightarrow b_1 = 0$

substituting a_1, b_1, c_1 in π_2

$$\pi_2 = H^{-1}\, g^0\, \rho^0\, D$$

$$\pi_2 = \frac{D}{H}$$

Third π-term:

substituting the dimensions on both sides.

$$M^0 L^0 T^0 = L^{a_3} (LT^{-2})^{b_3} (ML^{-3})^{c_3}\, ML^{-1}T^{-1}$$

Equating powers of M, L, T on both sides,

Power of M, $0 = c_3 + 1 \Rightarrow c_3 = -1$

Power of L, $0 = a_3 + b_3 - 3c_3 - 1 \Rightarrow 0 = a_3 - \frac{1}{2} + 2 \Rightarrow a_3 = -\frac{3}{2}$

Power of T, $0 = -2b_3 - 1 \Rightarrow b_3 = -\frac{1}{2}$

Substituting the values of a_3, b_3 and c_3 in π_3

$$\pi_3 = H^{-3/2}\, g^{-1/2}\, \rho^{-1}\, \mu$$

$$= \frac{\mu}{H^{3/2}\sqrt{g}\ \rho}$$

$$= \frac{\mu}{H\sqrt{H}\sqrt{g}\,\rho} \Rightarrow \frac{\mu}{H\rho\sqrt{gH}} \Rightarrow \frac{\mu}{HV\rho}\cdot\frac{V}{\sqrt{gH}}$$

$$\pi_3 = \frac{\mu}{HV\rho}\cdot\pi_1$$

Substituting π_1, π_2 and π_3 in equation (2),

$$f_1\left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \pi_1\frac{\mu}{HV\rho}\right) = 0;$$

$$\frac{V}{\sqrt{gH}} = \phi\left[\frac{D}{H}, \pi_1\frac{\mu}{HV\rho}\right]$$

$\times\frac{1}{\sqrt{2}}$
$$\frac{V}{\sqrt{2gH}} = \phi\left[\frac{D}{H}, \frac{\mu}{HV\rho}\right]$$

$$V = \sqrt{2gH}\ \phi\left[\frac{D}{H}, \frac{\mu}{HV\rho}\right]$$

Multiplying by a constant does not change the character of π-terms.

7. The pressure difference Δp in a pipe of diameter D and length l due to turbulent flow depends on the velocity V, viscosity μ, density ρ and roughness K. Using Buckingham's π theorem obtain an expression for Δp.

Nov/Dec 2016

Solution: Given. $\Delta p = f(D, l, V, \mu, \rho, K)$

$$\Rightarrow f_1(D, l, V, \mu, \rho, K, \Delta p) = 0. \quad \text{———} (1)$$

Dimensions:

$\Delta p - ML^{-1}T^{-2}$

$D - L \qquad \mu - ML^{-1}T^{-1}$

$l - L \qquad \rho - ML^{-3}$

$V - LT^{-1} \qquad K - L$

No. of variables , $n = 7$

No. of fundamental dimensions, $m = 3$

No. of π- terms $= n - m = 7 - 3 = 4$.

Eqn. (1) can be written as $f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0$;

<u>π-terms:</u>

$$\pi_1 = D^{a_1} V^{b_1} \rho^{c_1} \Delta p$$

$$\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} \lambda$$

$$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} \mu$$

$$\pi_4 = D^{a_4} V^{b_4} \rho^{c_4} k$$

<u>Results:</u>

$$\pi_1 = \frac{\Delta p}{\rho V^2} \qquad \pi_3 = \frac{\mu}{D V \rho}$$

$$\pi_2 = \frac{l}{D} \qquad \pi_4 = \frac{K}{D}$$

$$\frac{\Delta p}{\rho V^2} = \phi\left[\frac{l}{D}, \frac{\mu}{D V \rho}, \frac{K}{D}\right]$$

PART-C

1. The aerodynamic drag of a new sports car is to be predicted at a speed of 50.0 mi/h at an air temperature of 25°C. Automotive engineers build a one-fifth scale model of the car to test in a wind tunnel. It is winter and the wind tunnel is located in an unheated building; the temperature of the wind tunnel air is only about 5°C. Determine how fast the engineers should run the wind tunnel in order to achieve similarity between the model and the prototype.

SOLUTION

We are to utilize the concept of similarity to determine the speed of the wind tunnel. Assumptions 1.Compressibility of the air is negligible (the validity of this approximation is discussed later). 2. The wind tunnel walls are far enough away so as to not interfere with the aerodynamic drag on the model car. 3. The model is geometrically similar to the prototype. 4.The wind tunnel has a moving belt to simulate the ground under the car, as in Fig. (The moving belt is necessary in order to achieve kinematic similarity everywhere in the flow, in particular underneath the car.)

Properties

For air at atmospheric pressure and at T = 25°C, ρ = 1.184 kg/m^3 and μ= 1.849x 10^{-5} kg/m.s. Similarly, at T = 5°C, ρ = 1.269 kg/m3 and μ = 1.754x 10^{-5} kg/m · s.

Since there is only one independent π in this problem, the similarity equation holds if $\pi_{2m} = \pi_{2\rho}$

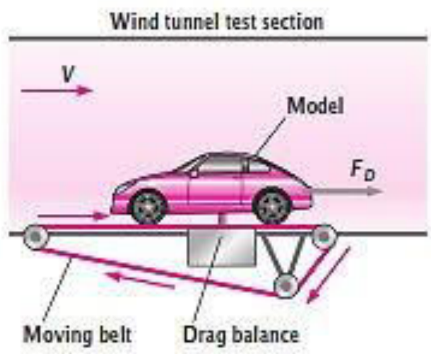
$$\Pi_{2,m} = \mathrm{Re}_m = \frac{\rho_m V_m L_m}{\mu_m} = \Pi_{2,p} = \mathrm{Re}_p = \frac{\rho_p V_p L_p}{\mu_p}$$

which can be solved for the unknown wind tunnel speed for the model tests, Vm,

$$Vm = Vp\ (\mu_m/\mu_p)(\rho_p/\rho_m)(L_p/L_m)$$

Substituting the values we have,

$\mathbf{V_m = 221\ m/h}$



Thus, to ensure similarity, the wind tunnel should be run at 221 mi/h (to three significant digits). Note that we were never given the actual length of either car, but the ratio of Lp to Lm is known because the prototype is five times larger than the scale model. When the dimensional parameters are rearranged as non-dimensional ratios (as done here), the unit system is irrelevant. Since the units in each numerator cancel those in each denominator, no unit conversions are necessary.

UNIT –V

TURBINES

1. **Define volumetric efficiency?**

    It is defined as the volume of water actually striking the buckets to the total water Supplied by the jet

2. **Write short notes on Draft tube?**

    It is a gradually increasing area which connects the outlet of the runner to the tail race. It is used for discharging water from the exit of the turbine to the tail race.

3. **How are hydraulic turbine classified?**

    1. According to the type of energy
    2. According to the direction of flow
    3. According to the head at inlet
    4. According to the specific speed of the turbine

4. **What is mean by hydraulic efficiency of the turbine?**

    It is ratio between powers developed by the runner to the power supplied to the water jet

5. **Define specific speed of the turbine**

    The speed at which a turbine runs when it is working under a unit head and develop unit power

6. **What is meant by governing of a turbine?**

    It is defined as the operation by which the speed of the turbine is kept constant under all conditions of working. It is done by oil pressure governor.

7. **List the important characteristic curves of a turbine**

    a. Main characteristics curves or Constant head curves
    b. Operating characteristic curves or Constant speed curves
    c. Muschel curves or Constant efficiency curves

8. Define gross head and net or effective head.

Gross Head: The gross head is the difference between the water level at the reservoir and the level at the tailstock.

Effective Head: The head available at the inlet of the turbine.

9. What is the difference between impulse turbine and Reaction turbine?

S.No	Reaction turbine	Impulse turbine
1.	Blades are in action at all the time	Blades are only in action when they are in front of nozzle
2.	Water is admitted over the circumference the wheel	Water may be allowed to enter a part or whole of the wheel circumference

10. Give example for a low head, medium head and high head turbine

Low head turbine – Kaplan turbine

Medium head turbine – Modern Francis

High head turbine – Pelton wheel

11. Explain the type of flow in Francis turbine?

The type of flow in Francis turbine is inward flow with radial discharge at outlet.

12. How do you classify turbine based on flow direction and working medium?

According to the direction of flow turbines are classified into

(i) Tangential flow turbine

(ii) Radial flow turbine

(iii) Axial flow turbine

(iv) Mixed flow turbine

According to the working medium turbines are classified into

(i) Gas turbine

(ii) Water turbine

(iii) Steam turbine

PART-B

1. A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at rate of 700 l/s. Under a head of 30 meters. The bucket deflect the jet through an angle 160°. Calculate power again by runner and hydraulic efficiency of turbine. Assume co-efficient of velocity as 0.98. (16)

[NOV/DEC – 2012]

Given:

$$U = U_1 = U_2 = 10\ m/s.$$

$$Q = 700\ l/s = 0.7\ m^3/s.$$

$$H = 30\,m$$

$$\phi = 180^\circ - 160^\circ = 20^\circ$$

$$C_v = 0.98.$$

Find:

(i) Power given to turbine $(P) = ?$

(ii) Hydraulic Efficiency of turbine $(\eta_h) = ?$

formula:

(i) $$\text{Power} = \frac{\text{Work done by the jet / second}}{1000}\ kW$$

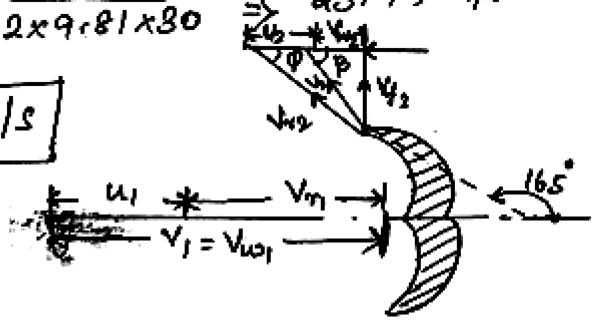
(ii) $$\text{Hydraulic Efficiency } (\eta_h) = \frac{2\,[V_{w_1} + V_{w_2}] \times U}{(V_1^2)}$$

Solution: (i) The velocity of jet $V_1 = C_v\sqrt{2gH}$.

$$V_1 = C_v\sqrt{2gH}.$$

$$= 0.98\sqrt{2\times9.81\times30} \Rightarrow 23.77 \text{ m/s}$$

$$\boxed{V_1 = 23.77 \text{ m/s}}$$



$$V_{r_1} = V_1 - u_1$$

$$V_{r_1} = 28.77 - 10 \Rightarrow 13.77 \text{ m/s}.$$

$$\boxed{V_{r_1} = 18.77 \text{ m/s}}$$

$$\boxed{V_{w_1} = V_1 = 23.77 \text{ m/s}}$$

From outlet velocity triangle,

$$V_{r_2} = V_{r_1} = 13.77 \text{ m/s}$$

$$V_{w_2} = V_{r_2}\cos\phi - u_2$$

$$= 13.77\cos 20^\circ - 10.0$$

$$\boxed{V_{w_2} = 2.94 \text{ m/s}}$$

(ii) Work done by the jet per second on the runner is given by equation.

$$= \rho a V_1 [V_{w_1} + V_{w_2}] \times u$$

$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$$

$$= 186970 \text{ Nm/s} \qquad [\because aV_1 = Q = 0.7 \text{ m}^3/\text{s}]$$

(iii) Power given to turbine $= \dfrac{\text{work done/sec}}{1000}$ kW

$$\Rightarrow \frac{186970}{1000}$$

$$\Rightarrow 186.97 \text{ kW}.$$

(iv) The hydraulic efficiency of the turbine (η_h) $= \dfrac{2[V_{w_1}+V_{w_2}]\times u}{(V_1)^2}$

$$\Rightarrow \frac{2[28.77 + 2.94]\times 10}{(28.77)^2}$$

$$\Rightarrow 0.9454 \text{ (or) } 94.54\,\%$$

Result :

(i) Power given to turbine (P) = 186.97 KW

(ii) The hydraulic efficiency of the turbine (η_h) = 94.54 %

2. In an Inward radial flow turbine, water enters at an angle of 22° to wheel tangent to outer rim and leaves at 3 m/s. Inner diameter 300mm & outer dia 600 mm. Speed is 300 rpm. The discharge through the runner radial.

Find the, (i) Inlet & outlet blade angles.
(ii) Taking Inlet width as 150mm. Find power developed by the turbine. (16)

[APr/ may-2010]

Given:

Guide blade angles $\alpha = 22°$.

Velocity of flow $V_{f_1} = V_{f_2} = 3$ m/s.

$D_1 = 300$ mm ; 0.3 m.

$D_2 = 600$ mm ; 0.6 m.

N = 300 rpm.

$\beta = 90°$ & $V_{w_2} = 0$

Inlet width $(B_1) = 150$ mm $= 0.15$ mm.

Find:
(i) Inlet & outlet blade angles.
(ii) Power developed by the turbine.

Formula :

(i) Inlet & outlet velocity triangles

Inlet velocity triangle $(\tan\theta) = \dfrac{V_{f_1}}{V_{w_1} - u_1}$

outlet velocity triangle $(\tan\phi) = \dfrac{V_{f_2}}{u_1}$

(ii) Power developed $(P) = \dfrac{\text{workdone per second}}{1000}$ KW.

Solu:

Tangential velocity of wheel at Inlet.

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 300}{60}$$

$$u_1 = 4.71 \text{ m/s}.$$

Tangential Velocity of wheel at outlet.

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 300}{60}$$

$$u_2 = 9.43 \text{ m/s}.$$

Absolute velocity of water at Inlet.

$$V_1 = \frac{V_{f_1}}{\sin\alpha} = \frac{3}{\sin 22} = 8.0084 \text{ m/s}.$$

Velocity of wheel at Inlet.

$$V_{w_1} = V_1 \cos\alpha = 8.0084 \times \cos 22$$

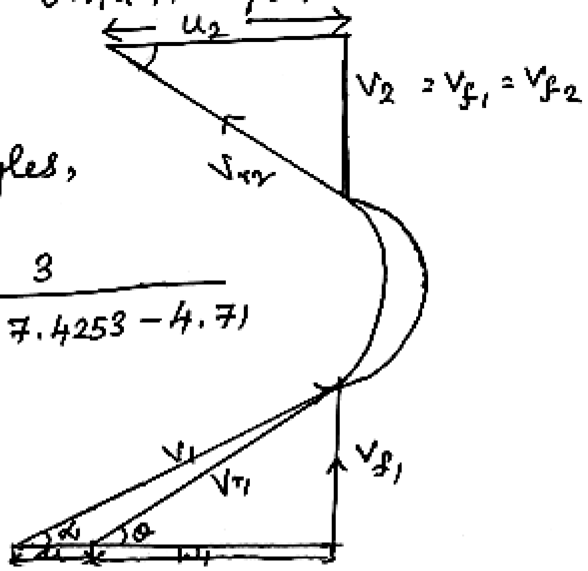
$$V_{w_1} = 7.4253 \text{ m/s}.$$

The Discharge $Q = \pi D_1 B_1 V_{f_1}$

$= \pi \times 0.3 \times 0.15 \times 3$

$= 0.4241\ m^3/s.$

For runner blade angles;



From Inlet velocity triangles,

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{3}{7.4253 - 4.71}$$

$\tan\theta = 1.1048$

$\theta = 47.85^\circ$

From outlet velocity triangles,

$$\tan\varphi = \frac{V_{f_2}}{u} = \frac{3}{9.43}$$

$= 0.3181.$

$\varphi = \tan^{-1}(0.3181)$

$\varphi = 17.65^\circ.$

Power developed,

$$P = \frac{\rho Q (V_{w_1} \times u_1)}{1000}$$

$$= \frac{1000 \times 0.4241 (7.4253 \times 4.71)}{1000}$$

$$\boxed{P = 14.83\ KW}$$

Result :

(i) Inlet velocity triangle θ $\}$ $= 47.85^\circ$

Outlet velocity triangle φ $= 17.65^\circ$

(ii) Power developed $(P) = 14.83\ KW.$

3. A Kaplan turbine working under a head of 20m develops 15 MW brake. The hub diameter 1.5m. runner diameter is 4m. The guide blade angle $\eta_h = 0.9$ & $\eta_o = 0.8$ Find runner vane angles & turbine speed. [16] [Apr/may - 2010]

Solution:

$H = 20\,m.$

$P = 15\,MW = 15000\,KW.$

$D_o = 4m.$

$D_b = 1.5\,m$

$\alpha = 30^\circ$

$\eta_h = 0.9 = 90\%.$

$\eta_o = 0.8 = 80\%.$

$\beta = 90^\circ$ & $V_{w_2} = 0$

Find :

$\theta = ?$

vane angles $\phi = ?$

turbine speed $N = ?$

Formula :

(i) $$\eta_o = \frac{\text{Shaft power}}{\text{water power}} = \frac{S.P}{\rho g Q H}.$$

$\eta_o = 0.80$

(ii) Vane angles $$\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$$

$$\tan\phi = \frac{V_{f_2}}{u_2} \; ; \; \tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

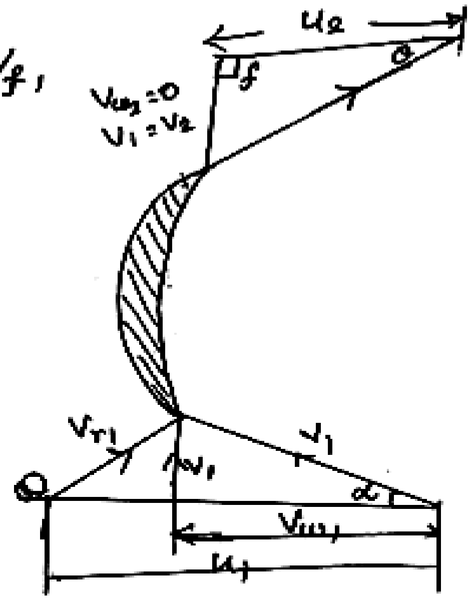
(iii) Speed of the turbine $N = ?$

Solution:

$$\eta_o = \frac{S.P}{\rho g Q H}$$

We know than

$$Q = \frac{\pi}{4}(D_a^2 - D_b^2) \times V_{f_1}$$



$$95.56 = \frac{\pi}{4}(4^2 - 1.5^2) \times V_{f_1}$$

$$\boxed{V_{f_1} = 8.8487 \text{ m/s}}$$

From Inlet velocity triangle

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$$

$$\tan 30^\circ = \frac{8.8487}{V_{w_1}}$$

$$\boxed{V_{w_1} = 15.33 \text{ m/s}}$$

Hydraulic Efficiency $\eta_h = \frac{V_{w_1} u_1}{gH}$

$$0.9 = \frac{15.33 \times u_1}{9.81 \times 20.}$$

$$\boxed{u_1 = 11.518 \text{ m/s}}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

$$= \frac{8.8487}{15.33 - 11.51}$$

$$= 2.3216$$

$$\tan\theta = 2.3216$$
$$\theta = \tan^{-1}(2.3216)$$
$$= 66.69$$
$$\boxed{\theta = 66.69^\circ}$$

For Kaplan turbine,

$$u_1 = u_2 = 11.518 \text{ m/s}$$
$$V_{f_1} = V_{f_2} = 8.8487 \text{ m/s}$$
$$\tan\varphi = \frac{V_{f_2}}{u_2} = 0.7682.$$
$$\varphi = \tan^{-1}(0.7682) = 37.53.$$

$$u_1 = \frac{\pi D N}{60}.$$
$$11.51 = \frac{\pi \times 4 \times N}{60}$$
$$\boxed{N = 54.997 \text{ rpm.}}$$

Result:

$$\theta = 66.69^\circ$$
$$\varphi = 37.53^\circ$$
$$N = 54.997 \text{ rpm.}$$

4. A Francis turbine developing 16120 KW under a head of 260 m runs at 600 rpm. The runner outside diameter is 1500 mm & the width is 135 mm. The flow rate is $7 m^3/s$. The exit velocity at the draft tube outlet is 16 m/s. Assuming zero whirl velocity at exit and neglecting blade thickness, determine the overall & hydraulic efficiency & rotor blade angle at Inlet. Also find the guide vane outlet angle. (16) [Nov/Dec -2014]

Given:

$$P = 16120 \text{ KW} \; ; \; H = 260 m \; ; \; N = 600 \text{ rpm}.$$

$$D_2 = 1.5 m \; ; \; B = 0.135 m \; ; \; Q = 7 m^3/s.$$

$$V_2 = V_{f_2} = 16 m/s \; ; \; V_{w_2} = 0.$$

To Find:

$$\eta_0 = ? \; ; \; \eta_h = ?$$

$$\alpha = ? \; ; \; \varphi = ?$$

Solution:

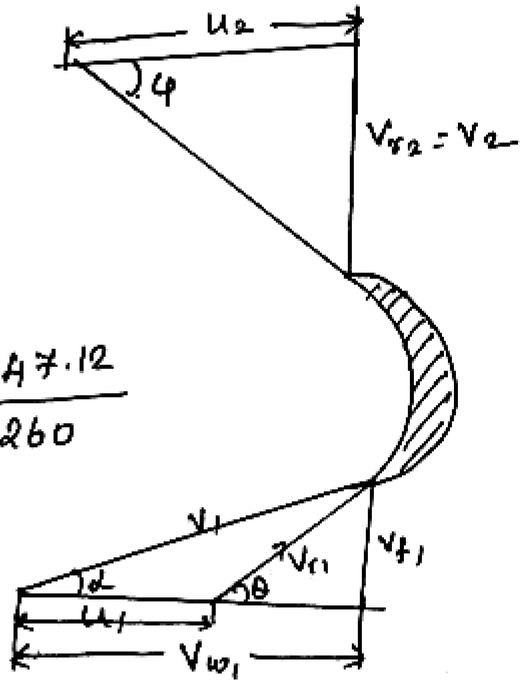
$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.5 \times 600}{60}$$

$$= 47.12 \text{ m/s}.$$

$$\text{Power developed } (P) = \frac{\rho Q V_{w_1} u_1}{1000}$$

$$\therefore 16120 = \frac{1000 \times 7 \times V_{w_1} \times 47.12}{1000}$$

$$\boxed{V_{w_1} = 48.86 \text{ m/s}}$$



$$\eta_h = \frac{V_{w1} u_1}{gH}$$

$$= \frac{48.86 \times 47.12}{9.81 \times 260}$$

$$\boxed{\eta_h = 0.902}$$

$$\boxed{\eta_h = 90.2\%}$$

$$\eta_o = \frac{S.P}{W.P} = \frac{S.P}{\rho g Q H}$$

$$= \frac{16120}{1000 \times 9.81 \times 7 \times 260}.$$

$$\boxed{\eta_o = 90\%}$$

$$\tan\alpha = \frac{V_{f1}}{V_{w1}}$$

$$Q = \pi D_1 B_1 V_{f1}$$

$$7 = \pi \times 1.5 \times 0.135 \times V_{f1}$$

$$\boxed{V_{f1} = 11\,m/s}$$

$$\tan\alpha = \frac{11}{48.86} = 0.225$$

$$\alpha = \tan^{-1}(0.225) = 12.68^\circ$$

$D_1 = 2 D_2$ (Assume most of the case)

$$D_2 = \frac{1.5}{2} = 0.75$$

$$u_2 = \frac{\pi D N}{60} = \frac{\pi \times 0.75 \times 600}{60} = 23.56 \text{ m/s}$$

$$\tan\varphi = \frac{18}{23.56} = 0.679$$

$$\varphi = \tan^{-1}(0.679) = 34^\circ 18' \quad \boxed{\varphi = 34^\circ 18'}$$

Result:

$\eta_o = 90\%$; $\eta_h = 90.2\%$; $\alpha = 12.68'$

$\varphi = 34^\circ 18'$

5. With a neat sketch, explain the construction and working of Pelton wheel.

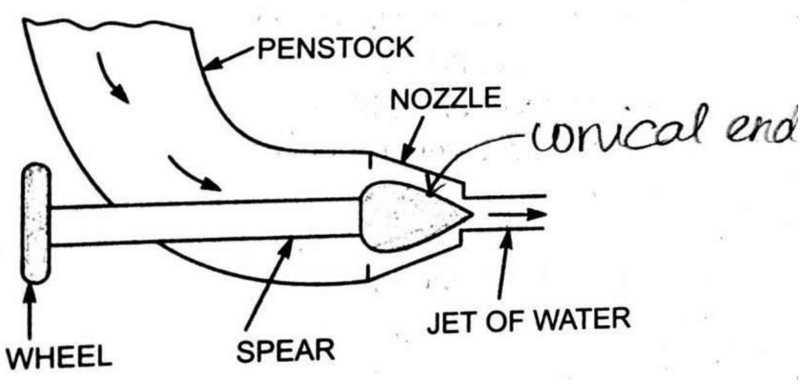
Pelton turbine is a tangential flow impulse turbine. It is named after L.A.Pelton, an American engineer. This turbine is used for high heads.

MAIN PARTS:

1. Nozzle and flow regulating valve
2. Runner and buckets
3. Casing
4. Breaking jet

1. Nozzle and flow regulating valve

The nozzle increases the kinetic energy of water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of jet and strikes the bucket of the runner. The amount of water striking the buckets of the runner is controlled by providing a spear in the nozzle. The spear is a conical needle which can be operated manually. When the spear is pushed forward or backward into



the nozzle the amount of water striking the runner is reduced or increased.

2. Runner and buckets

The runner consists of a circular disc with a number of bucket evenly spaced round its periphery. The shape of the bucket is of semi ellipsoidal cups. Each bucket is divided into two symmetrical parts by a dividing which is known as splitter. The splitter divides the jet into two equal parts and the jet comes out at the outer edge of the bucket.

The bucket is made up of cast iron, cast steel bronze or stainless steel depending upon the head at the inlet of the turbine.

3. Casing:

The function of casing is to prevent the splashing of the water and to discharge water to tail race. It also acts as a safeguard against accident.

It is made up of cast iron or fabricated steel plates.

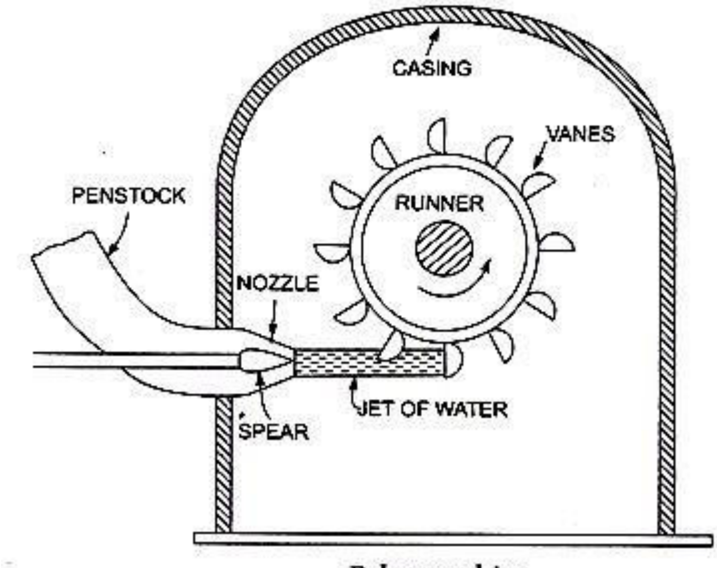
4. Breaking jet:

When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of the vanes. This jet of water is called breaking jet.

Working:

The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle increases the kinetic energy of water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of jet and strikes the bucket of the runner.

The water flows along the tangent to the path of rotation of the runner. The runner revolves freely in air. The water is in contact with only a part of the runner at a time, and throughout its action on the runner and in its subsequent flow to the tail race, the water is at atmospheric pressure. Casing is to prevent the splashing of the water and to discharge water to tail race.



Pelton turbine.

6. Draw the characteristic curves of the turbines. Explain the significance?

Characteristics curves of a hydraulic turbine are the curves, with the help of which the exact behavior and performance of the turbine under different working conditions can be obtained. These curves are plotted from the results of the tests performed on the turbine.

The important parameters which are varied during a test on a turbine:

1. Speed (N) 2.Head (H) 3. Discharge (Q) 4.Power (P)

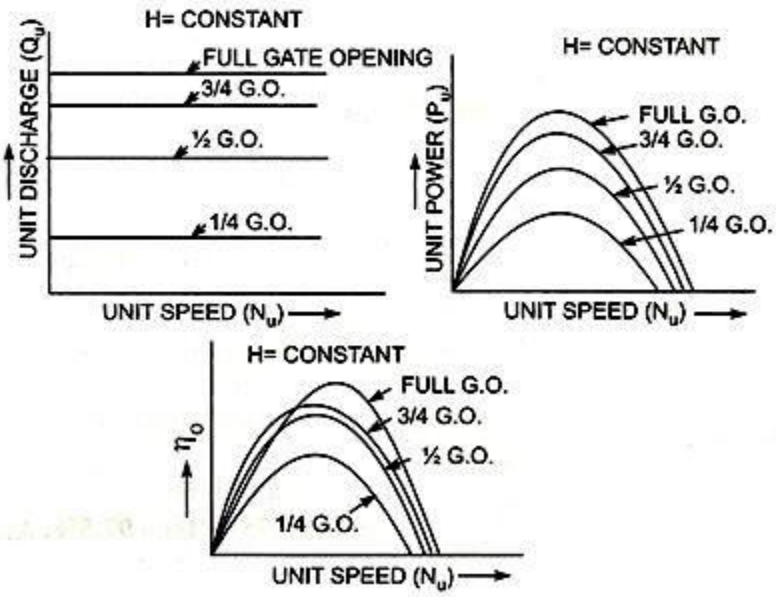
5. Overall deficiency(η_o) 6. Gate opening

Speed (N), Head (H), Discharge (Q) are independent parameters. One of the parameters are kept constant and the variation of the other four parameters with respect to any one of the remaining two independent variables are plotted and various curves are obtained. These curves are called characteristics curves. The following are the important characteristic curves of a turbine.

1. Main characteristics curves or constant head curves.
2. Operating characteristics curves or constant speed curves
3. Muschel curves of constant efficiency curves

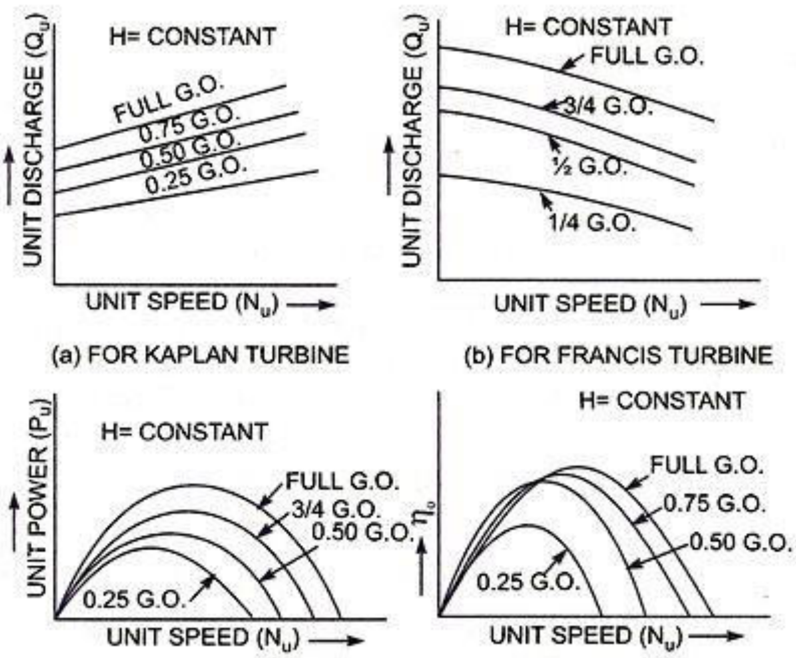
MAIN CHARACTERISTICS CURVES OR CONSTANT HEAD CURVES.

Main characteristics curves are obtained by maintaining a constant head and a constant gate opening on the turbine. The speed of the turbine is varied by changing load on the turbine. For each value of the speed, the corresponding values of the power (P) and discharge(Q) are obtained. Then the overall efficiency (η_o) for each value of the speed is calculated. From these readings the values of unit speed (N_u), unit power (P_u),and unit discharge (Q_u) are determined. **Main characteristics curves of a Pelton wheel as shown below.**



Main characteristic curves for a Pelton wheel.

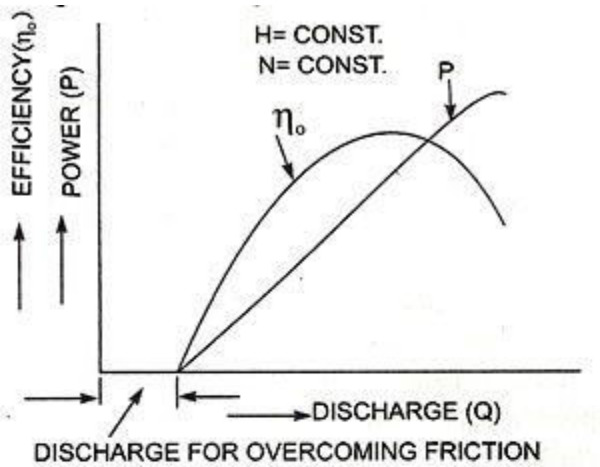
Main characteristics of a Kaplan and reaction turbine as shown below.



Main characteristic curves for reaction turbine.

OPERATING CHARACTERISTICS CURVES OR CONSTANT SPEED CURVES :

Operating Characteristics Curves are plotted when the speed on the turbine is constant. There are three independent parameters namely N, H and Q. For operating characteristics N and H are constant and hence the variation of power and efficiency with respect to discharge Q are plotted. The power curve for turbines shall not pass through the origin because certain amount of discharge is needed to produce power to overcome initial friction. Hence the power and efficiency curves will be slightly away from the origin on the x-axis as to overcome initial friction certain amount of discharge will be required.



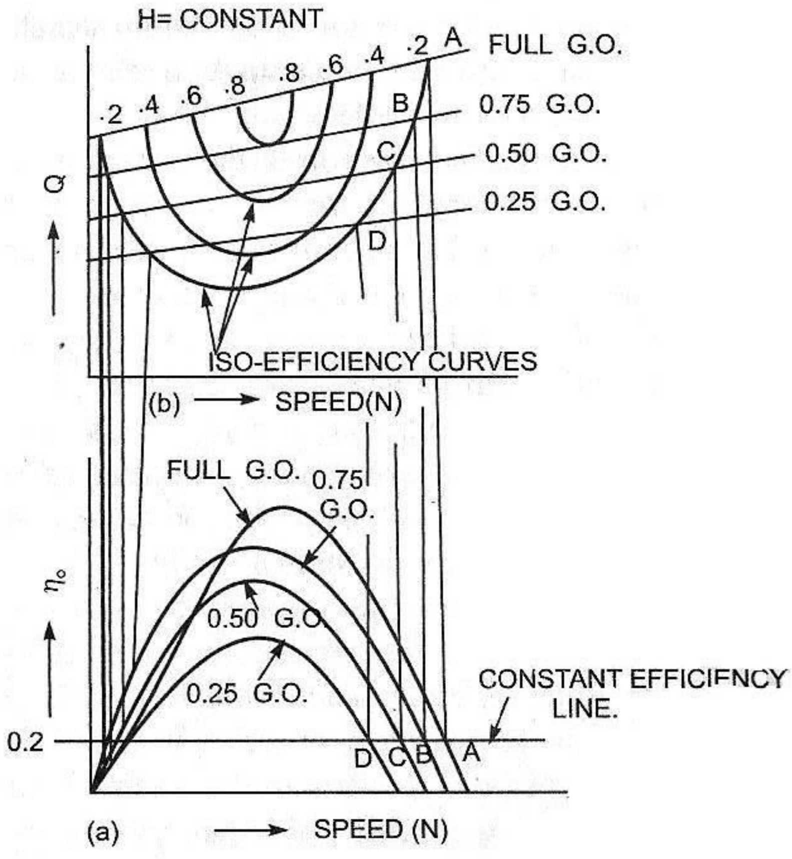
Operating characteristic curves.

MUSCHEL CURVES OF CONSTANT EFFICIENCY CURVES:

These curves are obtained from the speed Vs efficiency and speed Vs discharge curves for different gate openings. For a given efficiency, from the N_u vs η_o. Curves, there are two speeds. From the N_u vs Q_u curves, corresponding to two values of speeds there are two values of discharge. If the efficiency is maximum there is only one value. These two values of speed and two values of discharge corresponding to a particular gate opening are plotted.

The procedure is repeated for different gate opening and the curve Q vs N are plotted. The points having the same efficiency are iso-efficiency curves. These curves are useful to determine the zone of constant efficiency and for predicting the performance of the turbine at various efficiencies.

Horizontal lines representing the same efficiency are drawn on the η_o speed curves. The points at which these lines cut the efficiency curves at various gate opening are transferred to the corresponding Q- speed curves. The points having the same efficiency are then joined by smooth curves. These smooth curves represent the iso-efficiency curve.



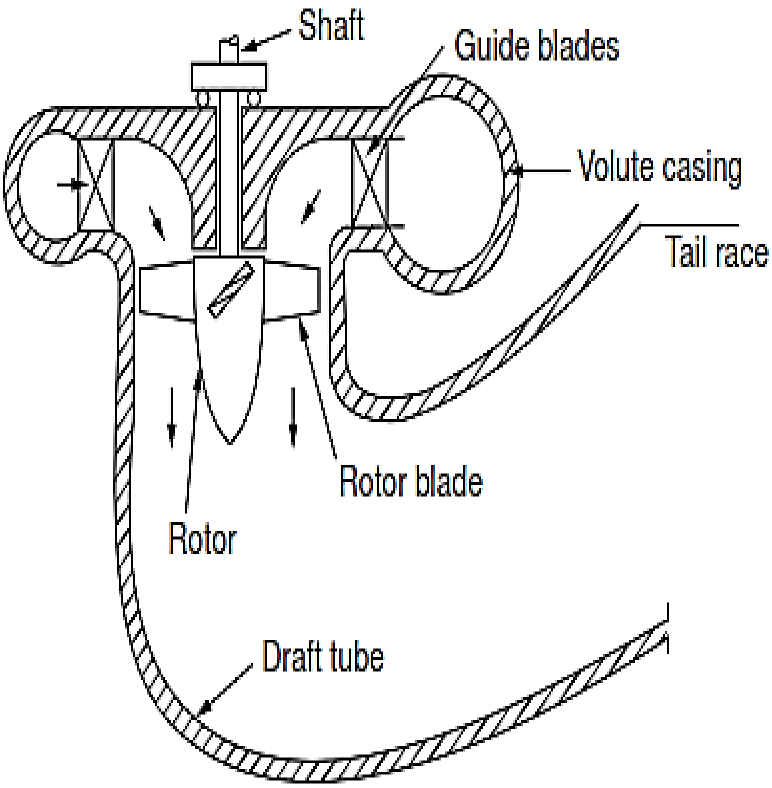
Constant efficiency curve.

7. Explain the working of Kaplan turbine. Construct its velocity triangles.

The popular axial flow turbines are the Kaplan turbine and propeller turbine. In propeller turbine the blades are fixed. In the Kaplan turbines the blades are mounted in the boss in bearings and the blades are rotated according to the flow conditions by a servo mechanism maintaining constant speed. In this way a constant efficiency is achieved in these turbines. The system is costly and where constant load conditions prevail, the simpler propeller turbines are installed. There are many locations where large flows are available at low head. In such a case the specific speed increases to a higher value. In such situations axial flow turbines are gainfully employed. A sectional view of a kaplan turbines in shown in figure. These turbines are suited for head in the range 5 – 80 m and specific speeds in the range 350 to 900. The water from supply pipes enters the spiral casing as in the case of Francis turbine. Guide blades direct the water into the chamber above the blades at the proper direction. The speed governor in this case acts on the guide blades and rotates them as per load requirements.

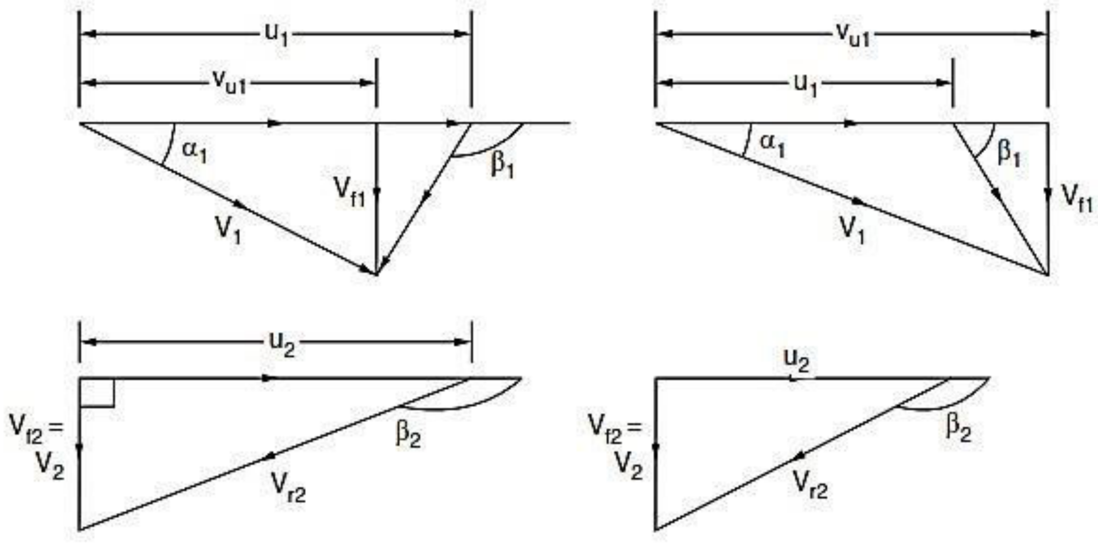
The flow rate is changed without any change in head. The water directed by the guide blades enters the runner which has much fewer blades (3 to 10) than the Francis turbine. The blades are also rotated by the governor to change the inlet blade angle as per the flow direction from the guide blades, so that entry is without shock. As the head is low, many times the draft tube may have to be elbow type. The important dimensions are the diameter and the boss diameter which will vary with the chosen speed. At lower specific speeds the boss diameter may be higher.

The number of blades depends on the head available and varies from 3 to 10 for heads from 5 to 70 m. As the peripheral speed varies along the radius (proportional to the radius) the blade inlet angle should also vary with the radius. Hence twisted type or Airfoil blade section has to be used.



Sectional view of Kaplan turbine

Velocity triangles



(*a*) At Tip (*b*) At Hub

PART-C

1. The head available at a location was 1500 m. It is proposed to use a generator to run at 750 rpm. The power available is estimated at 20,000 kW. Investigate whether a single jet unit will be suitable. Estimate the number of jets and their diameter. Determine the mean diameter of the runner and the number of buckets.

Solution:

The specific speed is calculated to determine the number of jets,

$$N_s = \frac{750}{60} \frac{\sqrt{20,000 \times 10^3}}{1500^{5/4}}$$

$$N_s = 5.99$$

So a single jet will be suitable.

The overall efficiency is assumed as 0.87.

$$20,000 \times 10^3 = 0.87 \times Q \times 1000 \times 9.81 \times 1500$$

$$\Rightarrow \boxed{Q = 1.56225 \ m^3/s}$$

To determine the jet velocity, the value of C_v is required. It is assumed as 0.97.

$$V = 0.97\sqrt{2gH}$$

$$= 0.97\sqrt{2 \times 9.81 \times 1500}$$

$$\boxed{V = 166.4 \text{ m/s}}$$

We know, $Q = A.V$

$$1.56225 = \frac{\pi}{4} d^2 \times 166.4$$

$$\Rightarrow \boxed{d = 0.1093 \text{ m}}$$

Assume, $\phi = 0.46$

$$u = 166.4 \times 0.46$$

Also, $u = \frac{\pi D N}{60}$

$$\Rightarrow D = \frac{60 u}{\pi N}$$

$$= \frac{60 \times 166.4 \times 0.46}{\pi \times 750}$$

$$\boxed{D = 1.95 \text{ m}}$$

Number of buckets, $= z. \frac{D}{2d} + 15$

$$= \frac{1.95}{2 \times 0.1093} + 15$$

$$= 24$$

2. **At a location selected to install a hydroelectric plant, the head is estimated as 550 m. The flow rate was determined as 20 m 3m/s. The plant is located at a distance of 2 m from the entry to the penstock pipes along the pipes. Two pipes of 2 m diameter are proposed with a friction factor of 0.029. Additional losses**

amount to about 1/4th of frictional loss. Assuming an overall efficiency of 87%, determine how much single jet unit running at 300 rpm will be required.

Solution:

Specific speed

Net head = Head available − loss in head

$$\text{friction Loss} = \frac{fLV_p^2}{2gD}$$

$Q = V_p \times A_p \times$ number of pipes

$Q = 20 m^3/s$ (given).

$$\Rightarrow V_p = \frac{20}{(\frac{\pi}{4} \times 2^2) \times 2} = 3.183 \, m/s$$

$V_p = 3.183 \, m/s$

$L = 2000 m$, $f = 0.029$

$$h_f = \frac{0.029 \times 2000 \times 3.183^2}{2 \times 9.81 \times 2}$$

$$\boxed{h_f = 14.98 \, m}.$$

$$\text{Total loss of head} = (1 - \frac{1}{4}) \times 14.98$$

$$= \frac{5}{4} \times 14.98$$

$$= 18.72 m$$

$\therefore$ Net head $= 550 - 18.72$

$= 531.28 m$

$\therefore$ Power, $P = \eta Q \rho g H$

$P = 0.87 \times 20 \times 1000 \times 9.81 \times 531.28$

$$\boxed{P = 90.6863 \times 10^3 W}$$

$$\text{specific speed, } N_s = \frac{300}{60} \cdot \frac{\sqrt{90.686 \times 10^3}}{531.28^{5/4}}$$

$$\boxed{N_s = 18.667}$$

Suitability of single jet unit

$$V_j = C_v\sqrt{2gH}$$

$$= 0.98\sqrt{2\times 9.81\times 531.28}$$

Velocity of jet, $V_j = 100.05$ m/s

Discharge, $Q = A \cdot V_j$

$$= \frac{\pi}{4} d^2 \times V_j$$

$$d = \left(\frac{4Q}{\pi V_j}\right)^{1/2}$$

$$d = \left(\frac{4\times 20}{\pi \times 100.05}\right)^{1/2}$$

$d = 0.5$ m (high)

Also, $\frac{\pi D N}{60} = 0.46 \times 100.05$

$D = 2.93$ m

Jet speed ratio $= \frac{2.95}{0.5}$

$= 6$ (low)

If three jets are suggested,

then $d = 0.29$ m

Jet speed ratio $= 10$ (Suitable).

$$\therefore N_s = \frac{300}{60}\sqrt{\frac{90.6863\times 10^6/3}{531.28^{5/4}}}$$

$N_s = 10.77$

Hence a three jet unit can be suggested.

U

UNIT –IV

PUMPS

1. **What is meant by Cavitations?**

It is defined phenomenon of formation of vapor bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapor pressure and the sudden collapsing of theses vapor bubbles in a region of high pressure.

2. **Define Slip of reciprocating pump. When the negative slip does occur?**

The difference between the theoretical discharge and actual discharge is called slip of the pump.

But in sometimes actual discharge may be higher than theoretical discharge, in such a case coefficient of discharge is greater than unity and the slip will be negative called as negative slip.

3. **What is meant by NSPH?**

Is defined as the absolute pressure head at the inlet to the pump, minus the vapour pressure head plus velocity head

4. **What is indicator diagram?**

Indicator diagram is nothing but a graph plotted between the pressure head in the cylinder and the distance traveled by the piston from inner dead center for one complete revolution of the crank

5. **What are rotary pumps?**

Rotary pumps resemble like a centrifugal pumps in appearance. But the working method differs. Uniform discharge and positive displacement can be obtained by using these rotary pumps, It has the combined advantages of both centrifugal and reciprocating pumps.

6. **What is meant by Priming?**

The delivery valve is closed and the suction pipe, casing and portion of the delivery pipe upto delivery valve are completely filled with the liquid so that no air pocket is left. This is called as priming.

7. **Define speed ratio, flow ratio**

Speed ratio: It is the ratio of peripheral speed at outlet to the theoretical velocity of jet corresponding to manometric head.

Flow ratio: It is the ratio of the velocity of flow at exit to the theoretical velocity of jet corresponding to manometric head.

8. **Mention the main parts of the centrifugal pump.**

(Nov/Dec 12)

1. Impeller
2. Casing
3. Suction pipe with foot valve and a strainer
4. Delivery pipe

9. **What is an air vessel? What are its uses?**

It is a closed chamber containing compressed air in the top portion and liquid at the bottom of the chamber

Uses

To obtain a continuous supply of liquid at a uniform rate

To save a considerable amount of work in overcoming the frictional resistance in the suction pipe

10. **Specific speed of a centrifugal pump.**

It is defined as the speed of a geometrically similar pump which would deliver one cubic metre of liquid per second against a head of one metre.it is denoted by 'N_S'

•

PART-B

1. The cylinder bore diameter of a single-acting reciprocating pump is 150mm and it's stroke is 300mm. The pump runs at 50 rpm and lifts water through a height of 25m. The delivery pipe 22m long & 100 mm in diameter. find the discharge and the theoritical power required to run the pump. If actual discharge 4.2 l/s. Find the percentage of slip. (16) [Nov / Dec - 2012]
Also determine the acceleration head at the beginning & middle of the delivery stroke.

Given:

diameter (d) = 150 mm = 0.15m

Length of stroke (L) : 300mm = 0.3m

Speed (N) = 50 r.p.m.

Height (H) = 25m

Length of delivery pipe (l_d) = 22m

dia. of delivery pipe (d_d) = 100 mm = 0.1m

Actual discharge Q_{act} = 4.2 l/s = 0.0042 m³/s.

Find:

(i) Theoretical discharge (Q_{th})

(ii) Theoretical power (P)

(iii) percentage of slip (%)

Formula:

(i) $Q_{th} = \frac{ALN}{60}$ (iii) % of slip $= \left(\frac{Q_t - Q_a}{Q_t}\right) \times 100$

(ii) $P = \frac{\rho g Q_{th} \times H}{1000}$

Solution:

(i) Theoretical discharge (Q_{th})

$$Q_{th} = \frac{ALN}{60}$$

$$A = \frac{\pi}{4}(d^2)$$

$$= \frac{\pi}{4}(0.15)^2$$

$$= 0.1767\, m^2.$$

$$Q_{th} = \frac{0.1767 \times 0.3 \times 50}{60} \Rightarrow 0.0044175\ m^3/s$$

$$\boxed{Q_{th} = 4.417\ l/s.}$$

(ii) Theoretical Power (P) $= \frac{\rho g Q_{th} \times H}{1000}$ (or) $\frac{\left(\frac{\text{workdone}}{\text{sec}}\right)}{1000}$

$$= \frac{1000 \times 9.81 \times 0.00441 \times 25}{1000}$$

$$\boxed{\text{Power (P)} = 1.0833\ \text{KW}}$$

(iii) Percentage of slip (%) $= \left(\frac{Q_{th} - Q_{act}}{Q_{th}}\right) \times 100$

$$= \left(\frac{4.4175 - 4.2}{4.4175}\right) \times 100 \Rightarrow 4.92\ \%$$

$$\boxed{\%\ \text{of slip} = 4.92\ \%}$$

$Q_{th} = 4.417\ l/s$

$P = 1.0833\ KW$

% of slip = 4.92 %

(iv) Acceleration head at the beginning of delivery stroke

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r \cdot \cos\theta$$

$$a_d = \pi/4\ (0.1)^2 = 0.007854$$

$$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi \times 50}{60} \Rightarrow 5.236\ r/s$$

$$\boxed{\omega = 5.236\ \text{rad/s}}$$

$$r = L/2 = \frac{0.3}{2} \Rightarrow 0.15\ m$$

$$h_{ad} = \frac{22}{9.81} \times \frac{0.01767}{0.007854} \times 5.236^2 \times 0.15 \times \cos\theta$$

$$= 20.75 \times \cos\theta$$

At the begining of delivery stroke $\theta = 0°$ & hence $\cos\theta = 1$

$$\boxed{h_{ad} = 20.75 m} \qquad [\because \cos\theta = 1]$$

(v) Acceleration head at the middle of delivery stroke

$\theta = 90°$ and hence $\cos\theta = 0$

$\therefore h_{ad} = 20.75 \times 0$

$$\boxed{h_{ad} = 0}$$

Result:

$Q_{th} = 4.417\ l/s$ $\qquad$ h_{ad} at begining $= 20.75m$

$P = 1.0833 KW$ $\qquad$ h_{ad} at middle $= 0$

% of slip = 4.92%

2. A single acting reciprocating pump running at 50r.p.m delivers 0.01 m³/sec of water. The diameter of the piston is 20cm & stroke length 40cm. Determine.

(i) The Theoretical discharge of the pump

(ii) Co-efficient of discharge.

(iii) Slip of the pump. (16) [NOV/Dec - 2008]

Given:

Speed of the pump (N) = 50 r.p.m

Actual discharge $(Q_a) = 0.01 m^3/s$

Dia. of Piston (D) = 200mm = 0.20m

$$\text{Area } (A) = \frac{\pi}{4}(0.2)^2$$

$$= 0.0314 m^2$$

$$L = 400mm = 0.40m$$

Find:

(i) Theoretical discharge

(ii) Co-efficient of discharge

(iii) Slip of the pump.

Formula :

$$Q_t = \frac{ALN}{60}$$

$$C_d = \frac{Q_{act}}{Q_t}$$

$$\text{Slip} = Q_{th} - Q_{act}$$

(i) Theoretical discharge $(Q_{th}) = \frac{ALN}{60}$

$$= \frac{0.031416 \times 0.40 \times 50}{60}$$

$$= 0.01047 \text{ m}^3/\text{s}.$$

(ii) Co-efficient of discharge

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047} = 0.955.$$

(iii) Slip $= Q_{th} - Q_{act}$

$$= 0.01047 - 0.01$$

$$= 0.00047 \text{ m}^3/\text{s}$$

Result :

(i) Theoretical Discharge $(Q_{th}) = 0.01047 \text{ m}^3/\text{s}$

(ii) Coefficient of discharge $(C_d) = 0.955$

(iii) Slip of the reciprocating pump is $= 0.00047 \text{ m}^3/\text{s}$.

3. The Internal and external diameter of Impeller of a Centrifugal pump are 200mm & 400mm respectively. The Pump is running at 1200 rpm. The Vane angles of cylinder at Inlet and outlet are 20° & 30° respectively. The water enters Impeller radially & Velocity of flow is Constant. Determine Workdone by Impeller per unit weight of water (16)

[NOV/DEC - 2012]

Given:

$$D_1 = 200 \text{ mm} = 0.20 \text{ m}$$

$$D_2 = 400 \text{ mm} = 0.40 \text{ m}$$

$N = 1200$ rpm.

$\theta = 20^\circ$; $\varphi = 30^\circ$

Find:

(i) workdone by Impeller.

Formula:

$$W = \frac{1}{g} V_{w_2} u_2 .$$

Solution:

$\alpha = 90^\circ$ and $V_{w_1} = 0$

$\because V_{f_1} = V_{f_2}$.

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60}$$

$$\boxed{U_1 = 12.56 \text{ m/s}}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.40 \times 1200}{60}$$

$$\boxed{U_2 = 25.13 \text{ m/s}}$$

$$\tan\theta = \frac{V_{f_1}}{U_1} = \frac{V_{f_1}}{12.56}$$

$$V_{f_1} = 12.56 \times \tan 20^\circ$$

$$= 4.57 \text{ m/s}$$

$$\boxed{V_{f_1} = V_{f_2} = 4.57 \text{ m/s}}$$

$$\tan\varphi = \frac{V_{f_2}}{U_2 - V_{w_2}} = \frac{4.57}{25.13 - V_{w_2}}$$

$$25.13 - V_{w_2} = \frac{4.57}{\tan\varphi}$$

$$25.13 - V_{w_2} = \frac{4.57}{\tan 30}$$

$$V_{w_2} = 25.13 - 7.915$$

$$\boxed{V_{w_2} = 17.215 \text{ m/s}}$$

Work done by Impeller,

$$W = \frac{1}{g} V_{w_2} U_2$$

$$= \frac{17.215 \times 25.13}{9.81}$$

$$= 44.1 \text{ Nm/s}$$

Result:

Workdone by the Impeller = 44.1 Nm/s.

4. A centrifugal pump delivers water against a net head of 14.5 meters and a design speed of 1000 rpm. The Vanes are curved back to an angle of 30° with the periphery. The Impeller diameter is 300 mm and outlet width 50 mm. Determine the discharge of the pump if Manometric Efficiency is 95%.

Given:

Net head $(H_m) = 14.5$ m

Speed $N = 1000$ r.p.m

Vane angle at outlet $\varphi = 30°$

Diameter $D_2 = 300$ mm $= 0.30$ m

Outlet width $B_2 = 50$ mm $= 0.05$ m

Manometric Efficiency. $\eta_{man} = 95\% = 0.95$

Find:

Discharge of the pump $(Q) = ?$

Formula:

$$Q = \pi D_2 B_2 \times V_{f_2}$$

Solution: Tangential velocity of Impeller at outlet $(U_2) = \frac{\pi D N}{60}$

$$= \frac{\pi \times 0.30 \times 1000}{60} \Rightarrow 15.70 \text{ m/s}$$

$$\boxed{U_2 = 15.70 \text{ m/s}}$$

$$\eta_{man} = \frac{g H_m}{V_{w_2} \times U_2}$$

$$0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70} \; ; \quad \boxed{V_{w_2} = 9.54 \text{ m/s}}$$

From outlet velocity triangle, we have

$$\tan\varphi = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow \tan 30^\circ = \frac{V_{f_2}}{(15.70 - 9.54)}$$

$$\tan 30^\circ = \frac{V_{f_2}}{6.16} \quad ; \quad \because V_{f_2} = 6.16 \times \tan 30^\circ$$

$$\boxed{V_{f_2} = 3.556 \text{ m/s}}$$

Discharge of the pump $(Q) = \pi D_2 B_2 \times V_{f_2}$

$$= \pi \times 0.30 \times 0.55 \times 3.55$$

$$= 0.1840 \text{ m}^3/\text{s}$$

Result:

Discharge of the centrifugal pump is $(Q) = 0.1840 \text{ m}^3/\text{s}$

5. The length & diameter of a suction pipe of a single acting reciprocating pump are 5m & 10cm respectively. The pump has a plunger of diameter 15cm & a stroke length of 35cm. The center of the pump is 3m above the water surface in the pump. The atmospheric pressure head is 10.3m of water and pump is running at 35 r.p.m (16)

[NOV/DEC-2011]

Determine,

(i) Pressure head due to acceleration at the begining of the suction stroke.

(ii) Max. Pressure head due to acceleration, and.

(iii) Pressure head in the cylinder at the beginning & at the end of the stroke.

Given:

Length of suction pipe $(l_s) = 5$m.

Dia. of suction pipe $(d_s) = 10$cm $= 0.1$m.

$$\therefore \text{Area } (a_s) = \frac{\pi}{4}(d_s)^2$$

$$= \frac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2$$

Dia of plunger $D = 15$cm $= 0.15$m.

$\therefore$ Area of Plunger $A = \frac{\pi}{4} D^2$

$$= \frac{\pi}{4} \times 0.15^2$$

$$= 0.01767 m^2$$

Stroke length, $L = 35 cm = 0.35 m$

$\therefore$ Crank radius $r = L/2$

$$= \frac{0.35}{2} = 0.175 m$$

Suction head $(h_s) = 3 m$

Atmospheric pressure head, $H_{atm} = 10.3$ m of water.

Speed $(N) = 35$ r.p.m.

Angular speed of the crank is.

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 35}{60}$$

$$\omega = 3.665 \text{ rad/s.}$$

(i) The pressure head due to acceleration in the suction pipe

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta.$$

At the beginning of stroke $\theta = 0°$ and hence $\cos\theta = 1$

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r$$

$$= \frac{5}{9.81} \times \frac{0.01767}{0.007854} \times 3.665^2 \times 0.175$$

$$\boxed{h_{as} = 2.695 m}$$

(ii) Max. pressure head due to acceleration in suction pipe

$$(h_{as})_{max} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r.$$

$$(h_{as})_{max} = \frac{5}{9.81} \times \frac{0.01767}{0.007854} \times 3.665^2 \times 0.175$$

$$= 2.695 m$$

(iii) Pressure head in the cylinder at the beginning of the suction stroke

$$= h_s + h_{as}$$

$$= 3 + 2.695 = 5.695.$$

This pressure head in the cylinder is below the atmospheric pressure head.

$\therefore$ Absolute pressure head in the cylinder at the beginning of suction stroke $= H_{atm} - h_{as}$

$$= 10.3 - 5.695$$

$$= 4.605 \text{ m of water (abs.)}$$

(iv) Similarly, The pressure head in the cylinder at the end of suction stroke.

$$= h_s - h_{as}$$

$$= 3 - 2.695 = 0.305 \text{ m}$$ which is below the atmospheric pressure head.

$\therefore$ Absolute pressure head in the cylinder at the end of suction stroke $= H_{atm} - h_{as}$

$$= 10.3 - 0.305$$

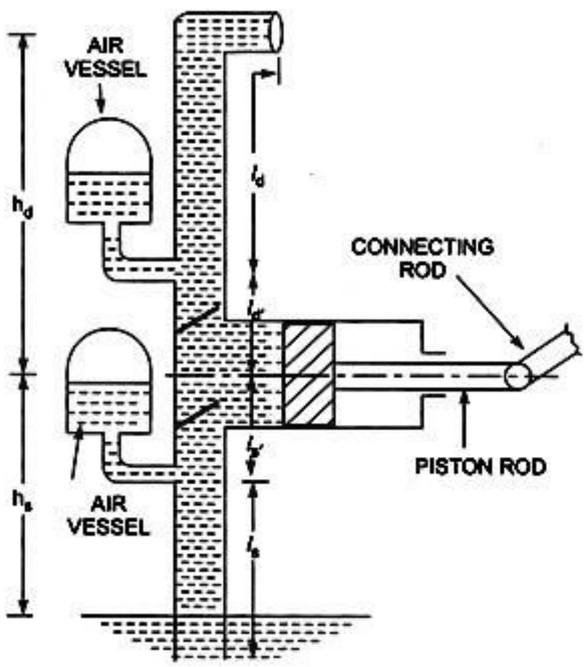
$$= 9.995 \text{ m of water (abs.)}$$

6(a) What is an air vessel? Describe the function of the air vessel for reciprocating pump with neat sketch. (8)

It is a closed chamber containing compressed air in the top portion and liquid (or water) at the bottom of the chamber. This is used to obtain a continuous supply of liquid at a uniform rate, to save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipes and to run the pump at high speed without separation.

The figure shows the single acting reciprocating pump to which air vessels are fitted to the suction and delivery pipes. The air vessels act like an

Intermediate reservoir. During the first half of the stroke, the piston moves with acceleration, which means the velocity of water in the suction pipe is more than



the mean velocity and hence the discharge of water entering the cylinder will be more than the mean discharge. This excess quantity of water will be supplied from the air vessel to the cylinder in such a way that the velocity in the suction pipe below the air vessel is equal to mean velocity of flow. During the second half of the suction stroke, the piston moves with retardation and hence the velocity of flow n the suction pipe is less than the mean velocity of flow. Thus, the discharge entering the cylinder will be less than the mean discharge. The velocity of water in the suction pipe due to air vessel is equal to mean velocity of flow and discharge required in cylinder is less than the mean discharge. Thus the excess water flowing in suction pipe will be stored into air vessel, which will be supplied during the first half of the stroke.

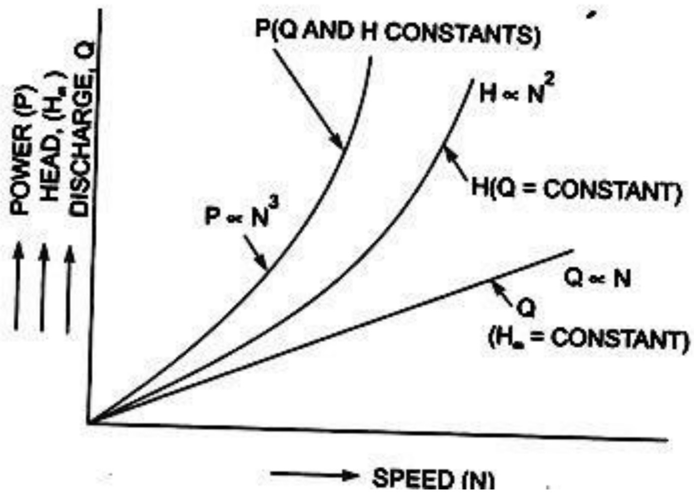
- During the second half of the delivery stroke, the piston moves with retardation and the velocity of water in the delivery pipe will be less than the mean velocity. The water already stored into the air vessel will start flowing into the delivery pipe and the velocity of flow in the delivery pipe beyond the point to which air vessel is fitted will become equal to the mean velocity. Hence the rate of flow of water in the delivery pipe will be uniform.

6(b) Draw and discuss the characteristic curves of centrifugal pumps. (8)

Main characteristic curves

The main characteristic curves of a centrifugal pump consists of variation of head H_m, power and discharge with respect to speed. For plotting curves of manometric head versus speed, discharge, is kept constant. For plotting curves of discharge versus speed, manometric head H_m is constant

For plotting the graph of H_m versus speed N, the discharge is kept constant. From equation $H \propto N^2$.this means that head developed by pump is proportional to the N^2 hence the curve is a parabolic curve. $P \propto N^3$. This means the curve is a cubic curve $Q \propto N$ hence it is a straight line.



Operating characteristic curves

If the speed is kept constant. The variation of manometric head, power and efficiency with respects to the discharge gives the operating characteristics of the pump.

The input curve for pumps shall not pass through the origin. It will be slightly away from the origin on the y-axis, as even at zero discharge some power is needed to overcome mechanical losses.

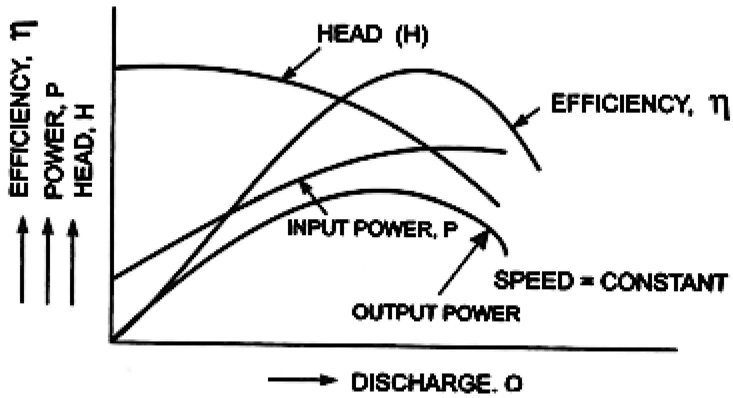
The head curve will have maximum value of head when discharge is zero.

The output power curve will start from origin as at Q=0, output power will be zero.

The efficiency curve will start from the origin as at $Q=0, \eta=0$

Constant Efficiency Curves

For obtaining constant efficiency curves for the pump, the head versus discharge curves and efficiency versus discharge curves for different speed are used. Fig shows the head versus discharge curves for different speeds. The efficiency versus discharge curves for the different speeds are as shown in Fig. by



combining these curves (H-Q curves and η –Q curves), constant efficiency curves are obtained

For plotting the constant efficiency curves (also known as iso -efficiency curves), horizontal lines representing constant efficiencies are drawn on the η-Q curves. The points, at which these lines cut the efficiency curves at various speed, are transferred to the corresponding H-Q curves. The points having the same efficiency are then joined by smooth curves. These smooth curves represent the iso efficiency curves.

7. Discuss the working of gear pump with its schematic (April/May 2017)

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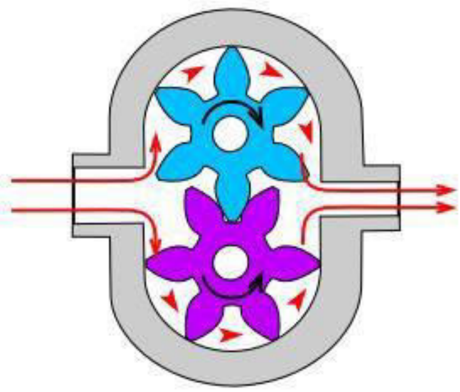

Gear pump-Schematic

Gear pump is a robust and simple positive displacement pump. It has two meshed gears revolving about their respective axes. These gears are the only moving parts in the pump. They are compact, relatively inexpensive and have few moving parts. The rigid design of the gears and houses allow for very high pressures and the ability to pump highly viscous fluids. They are suitable for a wide range of fluids and offer self- priming performance. Sometimes gear pumps are designed to function as either a motor or a pump. These pump includes helical and herringbone gear sets (instead of spur gears), lobe shaped rotors similar to Roots blowers (commonly used as superchargers), and mechanical designs that allow the stacking of pumps.

Construction:

One of the gears is coupled with a prime mover and is called as driving gear and another is called as driven gear. The rotating gear carries the fluid from the tank to the outlet pipe. The suction side is towards the portion whereas the gear teeth come out of the mesh. When the gears rotate, volume of the chamber expands leading to pressure drop below atmospheric value. Therefore the vacuum is created and the fluid is pushed into the void due to atmospheric pressure. The fluid is trapped between housing and rotating teeth of the gears. The discharge side of pump is towards the portion where the gear teeth run into the mesh and the volume decreases between meshing teeth. The pump has a positive internal seal against leakage; therefore, the fluid is forced into the outlet port. The gear pumps are often equipped with the side wear plate to avoid the leakage. The clearance between gear teeth and housing and between side plate and gear face is very important and plays an important role in preventing leakage. In general, the gap distance is less than 10 micrometers.