Reg. No. : E N G G T R E E . C O M

Question Paper Code : 51320

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024

Third Semester

Electrical and Electronics Engineering

MA 3303 – PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

(Normal distribution table values to be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State Baye's theorem.

2. Find the moment generating function of a random variable whose probability density function is given by $f(x) = \begin{cases} 1/k, & for 0 < x < k \\ 0, & otherwise \end{cases}$

3. If the joint probability density function of the random variable (X,Y) is given by $f(x,y) = cx(x-y)$, $0 < x < 2$, $-x < y < x$. Find the value of c.

4. The life time of a certain brand of an electric bulb may be considered a random variable with mean 1200 h and standard deviation 20 h. Find the probability, using central limit theorem, that the average lifetime of 0 bulbs exceeds 1250h.

5. Test the analyticity of the function $w = \sin z$.

6. Find the fixed points of the bilinear transformation $w = 2 - \frac{2}{z}$.

7. Make use of Cauchy's integral formula to find $\int_C \frac{z}{z-2}\, dz$, where c is $|z| = 3$.

8. Identify the type of singularity of function $f(z) = e^{\frac{1}{(z-1)}}$.

9. Solve $(D^2+1)y = e^{-x}$.

10. Build the equation $x^2 y'' + xy' = x$ into linear differential equation with constants coefficients.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) A bag contains 5 balls and is not known how many of the are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white? (8)

(ii) Find the probability density function of $Y = 8X^3$, given the random variable X with the density function $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$. (8)

Or

(b) (i) Out of 800 families with 4 children each, how many families would be expected to have. (8)

(1) 2 boys and 2 girls

(2) atleast 1 boy.

(ii) The time (in hour) to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. (8)

(1) What is the probability that the repair time exceeds 2h?

(2) What is the conditional probability that a repair takes at least 10 h given that its duration exceeds 9h?

12. (a) (i) The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}$, $0 \le x \le 2, 0 \le y \le 1$

Compute (8)

(1) $P(X > 1)$

(2) $P\left(Y < \frac{1}{2}\right)$

(ii) Find the coefficient of correlation between X and Y, using the following data: (8)

X:	1	3	5	7	8	10
Y:	8	12	15	17	18	20

Or

(b) (i) The probability density function (X,Y) is given by $f_{XY}(x,y) = x + y,\ 0 \le x,\ y \le 1$, find the probability density function of $U = XY$. (8)

(ii) Find the equations of the lines of regression from the following data : (8)

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

13. (a) (i) If $f(z) = u + iv$, is analytic find $f(z)$ if $u = \dfrac{2\sin 2x}{(e^{2y} + e^{-2y} - 2\cos 2x)}$. (8)

(ii) Show that $u = x^2 - y^2$ and $v = \dfrac{-y}{x^2 + y^2}$ are harmonic but $u + iv$ is not regular. (8)

Or

(b) (i) Find the bilinear map which maps the points $z = 1,\ i,\ -1$ onto $w = i,\ 0,\ -i$. (8)

(ii) Find the image of $|z - 2i| = 2$, under the transformation $w = \dfrac{1}{z}$. (8)

14. (a) (i) Evaluate, using Cauchy's integral formula $\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$. Where C is $|z| = 3$. (8)

(ii) Expand $\cos z$ as a Taylor's series about the point $z = \dfrac{\pi}{4}$. (8)

Or

(b) (i) Evaluate, the residue of $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ at each of the poles. (8)

(ii) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{13 + 12\cos\theta}$. (8)

15. (a) (i) Solve the equation $\left(D^3 - D^2 - 6D\right)y = x^2 + 1$. (8)

(ii) Solve $\left(D^2 + a^2\right)y = \tan ax$ by method of variation of parameters. (8)

Or

(b) (i) Solve the equation $(x+2)^2 \frac{d^2y}{dx^2} - (x+2)\frac{dy}{dx} + y = 3x + 4$. (8)

(ii) Solve the simultaneous equation $\frac{dx}{dt} + y = e^t$, $x - \frac{dy}{dt} = t$. (8)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 21277

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Third Semester

Electrical and Electronics Engineering

MA 3303 — PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

(Use the statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $P(C) = 0.65$, $P(D) = 0.40$ and $P(C \cap D) = 0.24$, Are the events C and D Independent?

2. Check whether the function $f(x) = \frac{x-2}{2}$ for $x = 1, 2, 3, 4$. Can serve as probability distributions.

3. If X_1 has mean 4 while X_2 has mean –2, find $E(2X_1 + X_2 - 5)$.

4. State the central limit theorem.

5. Let $f(z) = z^2 + 3z$. Find u and v and calculate the value of f at $z = 1 + 3i$.

6. Define conformal mapping.

7. Find the residue of $f(z) = \frac{\sin z}{z \cos z}$ at its pole inside the circle $|z| = 1$.

8. What type of singularities have the following function?

   (a) $f(z) = \frac{1}{1 - e^z}$ (b) $f(z) = \frac{e^{\frac{1}{z}}}{z^2}$

9. Are the function $e^{-x} \cos wx$ and $e^{-x} \sin wx$, $w \neq 0$ linearly independent? Justify.

10. Does the super position principal hold for non-homogeneous linear ODE? Justify.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) A first step towards identifying spam is to create a list of words that are more likely to appear in spam than in normal messages. For instance, words like buy or the brand name of an enhancement drug are more likely to occur in spam messages than in normal messages. Suppose a specified list of words is available and that your data base of 5000 messages contains 1700 that are spam. Among the spam messages, 1343 contain words in the list. Of the 3300 normal messages, only 297 contain words in the list. Obtain the probability that a message is spam given that the message contains words in the list. (8)

(ii) Verify that the functions $f(x) = \frac{2x+1}{25}, x = 0,1,2,3,4$ are probability mass functions, and determine the following: (8)

(1) $P(X = 4)$ (2) $P(X \leq 2)$

(3) $P(2 \leq X < 4)$ (4) $P(X \geq 2)$

Or

(b) (i) The random variable X has a binomial distribution with $n = 10$ and $p = 0.1$. Determine the following probabilities. (8)

(1) $P(X \leq 2)$ (2) $P(X > 8)$

(3) $P(X = 4)$ (4) $P(5 \leq X \leq 7)$

(ii) Suppose X has an exponential distribution with mean equal to 10. Determine the following: (8)

(1) $P(X > 10)$ (2) $P(X < 30)$

(3) Find the value of x such that $P(X < x) = 0.95$

12. (a) Determine the value of c that makes the function $f(x, y) = c(x + y)$ a joint probability mass function over the nine points with X = 1, 2, 3 and Y = 1, 2, 3. Determine the following: (16)

(i) $P(X = 5, Y < 4)$

(ii) $P(X = 1)$

(iii) $P(Y = 4)$

(iv) Marginal probability distribution of the random variable X

(v) Find correlation and covariance

Or

(b) (i) Determine the value of c such that the function $f(x,y)=cxy$, for $0<x<3$ and $0<y<3$ satisfies the properties of a joint probability density function. Determine the following: (8)

(1) $P(X<2, Y<3)$ (2) $P(X<2.5)$

(3) $E(X), E(Y)$ (4) $Var(X)$ and $Var(Y)$

(5) Conditional probability distribution of Y given that $X = 1.5$

(ii) Find a least square straight line for the following data (8)

X	1	2	3	4	5	6
Y	6	4	3	5	4	2

13. (a) (i) State and prove Cauchy-Riemann equations for analytic function. (8)

(ii) Verify that $u(x,y)=x^2-y^2-y$ is harmonic in the whole complex plane and find a harmonic conjugate function v of u. (8)

Or

(b) (i) Show that $w=\frac{i-z}{i+z}$, maps the real axis z-plane into the circle $|w|=1$. (8)

(ii) Find the bilinear transformation which maps the points $z=-1,i,1$ onto the points $w=0,i,\infty$ respectively. (8)

14. (a) State and prove Cauchy's integral formula. Use this formula to evaluate $\int_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$ where C is the circle $|z|=3$. (8+8)

Or

(b) (i) Find the Laurent's series expansion of $f(z)=\frac{7z-2}{z(z+1)(z-2)}$ in the region $1<z+1<3$. (8)

(ii) Evaluate the real integral $\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+1)(x^2+4)}$ by using Cauchy's residue theorem. (8)

15. (a) (i) Find the general solution of the Euler-Cauchy equations $x^2y'' + xy' + 16y = 0$. (6)

(ii) Solve by method of variation of parameters $y'' - 2y' + y = e^x \log x$. (10)

Or

(b) (i) Solve the simultaneous equations $\frac{dx}{dt} + 2y + \sin t = 0, \frac{dy}{dt} - 2x - \cos t = 0$ Given that $x = 0$ and $y = 1$ when $t = 0$. (8)

(ii) Solve by the method of undetermined coefficients $(D^2 + 1)y = \sin x$. (8)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 30239

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third Semester

Electrical and Electronics Engineering

MA 3303 — PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Note : Statistical table to be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the axioms of probability?

2. Give the moment generating function of Binomial and Poisson distributions.

3. Define Central limit theorem.

4. Define covariance of random variables X and Y. Define their independence using covariance.

5. What are the necessary conditions for a function to be analytic?

6. What is conformal mapping?

7. Show that $\oint_c \frac{dz}{z-a} = 2\pi i$, where '$a$' is any point within simple closed curve 'C'.

8. Find the nature of singularity of $f(z) = \frac{z - \sin z}{z^2}$.

9. Solve the differential equation $(D^2 + 5D + 6)y = 0$.

10. Why does the method of undetermined coefficients fail when trial solution is assumed as $X = \tan x$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A manufacturer of tablets receives its LED screens from three different suppliers B_1, B_2 and B_3. The probability that any one LED screen received by the plant comes from these three suppliers are, 0.60, 0.30 and 0.10 respectively. Suppose that 95% of the LED screens from B_1, 80% of those from B_2 and 65% of those from B_3 perform according to specifications. What is the probability that the LED screen received from any plant, performs according to specification. Also find the probability that LED screen working under specification has come from (1) Supplier B_1, (2) Supplier B_2, (3) Supplier B_3. (8)

(ii) Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, (8)

(1) Exactly 2 contain pollutant.

(2) Atleast four samples contain pollutant.

Or

(b) (i) The maximum attenuation occurring in the barcode scanner, changes from product to product. After collecting considerable data, the engineers decided to model the variation occurring as normal distribution with mean 10.1 dB and standard deviation 2.7 dB. For the next product what is the probability that its maximum attenuation is between 8.5 dB and 13.0 dB. Also what proportion of the products has maximum attenuation greater than 15.1 dB? (8)

(ii) In a large corporate network, the time interval between user log-ons to the system can be modelled as exponential distribution with mean 1/25 log-ons per hour. What is the probability that (8)

(1) There are no log-ons in an interval of six minutes?

(2) The time until next log-on is between two and three minutes?

(3) Determine the interval time such that the probability that no log-on occurs in the interval is 0.90.

12. (a) Determine the value of c that makes the function $f(x,y)=c(x+y)$, a joint probability mass function over the nine points with $x=1,2,3$ and $y=1,2,3$. (16)

(i) What is the marginal probability distribution of random variable X and Y?

(ii) What is the conditional probability distribution of Y given X = 1?

(iii) Find $P(X<2,Y<2)$, $P(X=1,Y<2)$

(iv) Find covariance (X, Y). Are X and Y independent?

Or

(b) An engineer conducts an experiment with the purpose of showing that adding a new component to the existing metal alloy increases the cooling rate. Let X denote the percentage of the new component present in the metal. Let Y denote the cooling rate, during a heat treatment stage in degree Fahrenheit per hour. The observed data are

X	0	1	2	2	4	4	5	6
Y	25	20	30	40	45	50	60	50

Fit a simple linear regression equation, for the given data. Estimate the value of cooling rate when new component percentage is 5.5%, using the fitted equation. Calculate the residuals and the error sum of squares of the fitted line. Estimate the correlation coefficient of the given data. (16)

13. (a) (i) If $f(z)$ is analytic function with constant modulus, show that $f(z)$ is constant. (8)

(ii) If $\omega=\phi+i\psi$ represents the complex potential for an electric field and $\psi=x^2-y^2+\frac{x}{x^2+y^2}$ determine the function. (8)

Or

(b) (i) Find the bilinear transformation which maps the points $z=1,i,-1$ onto the points $w=i,0,-i$ Hence find the image of $|z|<1$. (8)

(ii) Interpret the transformation of points from z-plane to w-plane by (1) Translation, (2) Rotation, (3) Inversion. (8)

14. (a) (i) State and prove Cauchy's integral theorem. (8)

(ii) Evaluate the integral $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$, using Cauchy's integral formula. (8)

Or

(b) (i) Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$, using Cauchy Residue integral. (8)

15. (a) (i) Solve the differential equation $(D^2 + D + 1)y = (1 - e^x)^2$. (8)

(ii) Using method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} + 4y = \tan 2x$. (8)

Or

(b) (i) Solve the Legendre's linear equation

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)].$$ (8)

(ii) Solve the simultaneous equations $\frac{dy}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$. (8)

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Reg. No. : E N G G T R E E . C O M

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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Electrical and Electronics Engineering

MA 3303 – PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Statistical Z table should be given.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A pair of dice is tossed twice. Find the probability of scoring 7 points at least once.

2. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that exactly two will be defective.

3. If the joint probability density function of the random variable (X, Y) is given by $f(x,y) = kxye^{-(x^2+y^2)}$, $x > 0$, $y > 0$. Find the value of K.

4. State central limit theorem.

5. Show that the function $f(z) = z\bar{z}$ is nowhere analytic.

6. Under the transformation $W = \frac{1}{Z}$, find the images of $2x + y = 2$.

7. Evaluate $\int_c \frac{e^z}{z(1-z)^3}\, dz$ if 0 lies inside c and 1 lies outside c.

8. Expand $f(z) = e^z$ as a Taylor's series about $z = 0$.

9. Solve $(D^2 - 3D + 2)y = e^x$.

10. Solve $(xD^2 + D)y = 0$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has the following probability function: (8)

x :	0	1	2	3	4	5	6	7
p(x) :	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(1) Find the value of k?

(2) Evaluate $P(X<6)$ and $P(0<X<5)$.

(ii) In 256 sets of 12 tosses of a coin in how many cases one can except 8 heads and 4 tails. (8)

Or

(b) (i) In attest on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for (8)

(1) More than 2150 hours,

(2) Less than 1950 hours.

(ii) Find the moment generating function of the exponential distribution $f(x)=\frac{1}{c}e^{-\frac{x}{c}}, 0 \le x \le 8, c>0$. Hence find its mean and standard deviation. (8)

12. (a) (i) The joint probability mass function of (X,Y) is given by $f(x,y)=k(2x+3y), x=0,1,2; y=1,2,3$. Find all the marginal distribution of X given $Y=2$, $Y=3$. (8)

(ii) Find the covariance for the following heights of fathers X and their sons Y: (8)

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

Or

(b) (i) A study of prices of rice of Chennai and Madurai gave the following data: (8)

	Chennai	Madurai
Mean	19.5	17.75
S.D.	1.75	2.5

Also the coefficient of correlation between the two is 0.8. Estimate the most likely price at Chennai corresponding to the price of 18 at Madurai.

(ii) If X and Y follow exponential distribution with parameter 1 and are independent, find the probability distribution function of $U = X - Y$. (8)

13. (a) (i) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

(ii) If $f(z)=u+iv$, is analytic find $f(z)$ and v if $u=\frac{\sin 2x}{\cos 2x+\cos h2y}$. (8)

Or

(b) (i) Find the bilinear map which maps the points $z=1, i, 0$ onto $w=1, i, -1$. (8)

(ii) Show that the map $w=\frac{1}{z}$ maps the totality of circles and lines as circles or lines. (8)

14. (a) (i) Evaluate, using Cauchy's integral formula $\int_c \frac{z+1}{z^2+2z+4}\,dz$, where c is the circle $|z+1+i|=2$. (8)

(ii) Expand $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ in a Laurent's series if $|z|<2$. (8)

Or

(b) (i) Evaluate $\oint_C \frac{z\sec z}{(1-z^2)}dz$, where c is the ellipse $4x^2+9y^2=9$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$. (8)

15. (a) (i) Solve the equation $(D^2-4D+3)y=\sin 3x+x^2$. (8)

(ii) Solve $(x^2D^2-xD+1)y=\left(\frac{\log x}{x}\right)^2$. (8)

Or

(b) (i) Solve the equation $\frac{d^2y}{dx^2}+a^2y=\tan ax$, by the method of variation of parameters. (8)

(ii) Solve the simultaneous equation $\frac{dx}{dt}+2x-3y=t, \frac{dy}{dt}-3y+2y=2e^{2t}$. (8)

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